Young Geometric Group Theory IX

Saint-Jacut-de-la-Mer
February 24-28, 2020

Introduction

Welcome to the ninth edition of the Young Geometric Group Theory meeting!

In this booklet, apart from the abstracts of the mini-courses and research talks, you will find the research statements of all the participants. We strongly encourage you to read through all of them.

In this way, you will get a panoramic view of the current subjects of research in the field of geometric group theory. You will surely discover subjects you never knew existed. Maybe, you will discover as well that someone else has some common interests with you. So, this could be the beginning of passionate scientific discussions!

This winter school is thought as a forum for sharing ideas in a friendly atmosphere. Feel free to ask the speakers, the organizers and other participants. In this way, you will certainly make the best of your week in Saint-Jacut-de-la-mer.

We wish you a pleasant and fruitful winter school!

The organizers.

Contents

Mini-courses

Serge Cantat
CNRS / Université de Rennes 1, France

Groups of polynomial transformations

Consider the affine space of dimension $k > 1$, say over the field of complex numbers, and the group of all its polynomial transformations with a polynomial inverse. By definition, this is the group of polynomial automorphisms of the affine space: it contains all affine transformations, but also non linear automorphisms of arbitrary large degrees. What properties of linear (finitely generated) groups remain valid in this larger group? This will be the main topic of the mini-course.

Anna Erschler
CNRS / ENS Paris, France

Growth and amenability of groups

Growth function counts the number of elements of word length at most n in the word metric. While many known groups have either polynomial or exponential growth, a rich and interesting class of groups of intermediate growth was discovered in the eighties by Grigorchuk. Despite some progress in understanding such groups in recents decades, there is still some mystery about these groups and many fundamental questions remain open. Amenable groups are groups admitting an invariant finitely additive measure defined on all their subsets. Equivalent definitions can be given in terms of isoperimetric inequalities in Cayley graphs and in terms of random walks. It is easy to see that any group of subexpenential growth is amenable. Moreover, it is known that any group of intermediate growth is amenable, but not elementary amenable. In my course I discuss known results and open questions about growth and amenability.

Peter Haïssinsky
Aix-Marseille Université, France

Convergence groups

Convergence groups form a class which appears naturally in different contexts. They are subgroups of homeomorphisms of Hausdorff compact spaces which are characterized by their dynamical properties. After going through their general properties, we will discuss some of their particular features. We may explore their relationships with hyperbolicity and quasiconformal maps, focus on groups acting on the 2-sphere and/or analyse their actions on compact subsets.

Nicolas Monod
École Polytechnique Fédérale de Lausanne, Switzerland

Spaces of the third kind

There are three model spaces: Euclidean, spherical and hyperbolic. In many branches of analysis and geometry, we use the first kind without specifying any dimension, and indeed often in infinite dimensions: this is called Hilbert spaces. Likewise, for the second kind, Hilbert spheres are very familiar since they underly unitary representations. In this mini-course, we study the third kind: Hyperbolic spaces of arbitrary (typically infinite) dimension. This is in particular interesting from the viewpoint of group actions.

Junior speakers

Federica Fanoni
CNRS / Université de Paris-Est Créteil, France

Big mapping class groups and their actions on graphs

Mapping class groups of finite-type surfaces (surfaces with finitely generated fundamental group) form an interesting class of finitely generated groups. If we consider infinite-type surfaces instead (surfaces whose fundamental group is not finitely generated), the associated mapping class groups are not finitely generated and thus not adapted to being investigated via classical geometric group theory techniques. I will discuss how one could still study these groups and show that in many cases they still act nicely on interesting graphs associated to the surface. Based on joint works with M. Durham and N. Vlamis and with T. Ghaswala and A. McLeay.

Mikołaj Frączyk
IAS Princeton, USA

Growth of mod-p homology groups in higher rank lattices.

Let Γ be either a higher rank lattice or finite index subgroup of the mapping class group of higher genus surface. I will describe two approaches for estimating the dimensions of mod-p homology groups in terms of the covolume or the index of Γ in the ambient group. First approach is homotopical in nature and involves building "small" CW-complexes computing low homology groups while the second in purely geometric and requires the understanding of minimal representatives of homology classes. Talk will be partly based on a joint work (in progress) with M. Abert, N. Bergeron and D. Gaboriau.

Radhika Gupta
University of Bristol, UK

Non-uniquely ergodic arational trees in the boundary of outer space

The mapping class group of a surface is associated to its Teichmüller space. In turn, its boundary consists of projective measured laminations. Similarly, the group of outer automorphisms of a free group is associated to its outer space. Now the boundary contains equivalence classes of arational trees as a subset. There exist distinct projective measured laminations that have the same underlying geodesic lamination, which is also minimal and filling. Such geodesic laminations are called “non-uniquely ergodic”. I will first talk about laminations on surfaces and then present a construction of non-uniquely ergodic phenomenon for arational trees. This is joint work with Mladen Bestvina and Jing Tao.

Yair Hartman
Ben Gurion University, Israel

Which groups have bounded harmonic functions?

Bounded harmonic functions on groups are closely related to random walks on groups. It has long been known that all virtually nilpotent groups are "Choquet-Deny groups": these groups cannot support non-trivial bounded harmonic functions. Equivalently, their Furstenberg-Poisson boundary is trivial, for any random walk. I will present a recent result where we complete the classification of discrete countable Choquet-Deny groups, proving a conjecture of Kaimanovich-Vershik. We show that any finitely generated group which is not virtually nilpotent, is not Choquet-Deny. Surprisingly, the key here is not the growth rate, but rather the algebraic infinite conjugacy class property (ICC). This is joint work with Joshua Frisch, Omer Tamuz and Pooya Vahidi Ferdowsi.

Adrien Le Boudec
CNRS/ENS Lyon, France

Commensurated subgroups and micro-supported actions

A subgroup Λ of a group Γ is commensurated if all the conjugates of Λ are commensurate. After providing a basic introduction to this notion, we will state a theorem that relates the commensurated subgroups of a finitely generated group Γ with the topological dynamics of the minimal and micro-supported actions of Γ on compact spaces. As an application we obtain a criterion to exclude the existence of non-trivial commensurated subgroups in certain classes of groups. Examples include topological full groups of amenable groups, or branch groups acting on rooted trees.

Although the theorem is about finitely generated discrete groups, we will try to highlight the role played in the proof by non-discrete locally compact groups. Time permitting, we might also discuss how the notion of uniformly recurrent subgroups (URS) comes into play. This is a joint work with Pierre-Emmanuel Caprace.

Beatrice Pozzetti
Universität Heidelberg, Germany

Surface subgroups of semisimple Lie groups

I will discuss various geometric properties of discrete subgroups of semisimple Lie groups G, isomorphic to the fundamental group of a hyperbolic surface. After discussing the differences between the case $G = \mathrm{SL}(2, \mathbb{R})$, where we will recover Teichmüller theory, and $G = \mathrm{SL}(2, \mathbb{C})$ where we will encounter fundamental groups of quasi-Fuchsian manifolds and their limits, I will explain how for $\mathrm{SL}(n, \mathbb{R})$, and more generally in higher rank, new interesting intermediate phenomena arise. Based on joint work with Sambarino and Wienhard and with Beyrer.

Research statements

Ilya Alekseev
Saint Petersburg State University, Russia

Geodesic words for finitely generated groups

Suppose G is a group with a finite generating set S. A word in the alphabet $S \cup S^{-1}$ is said to be geodesic if it has minimal length among all words representing the same element of G. While thinking about geodesic words, I imagine the shortest paths in the Cayley graph. In my research, I focus on geodesic words for mapping class groups and free nilpotent groups. My results deal with the braid groups

$$B_n = \left\langle \sigma_1, \ldots, \sigma_{n-1} \,\middle|\, \begin{array}{cr} \sigma_k\sigma_{k+1}\sigma_k = \sigma_{k+1}\sigma_k\sigma_{k+1}, & 1 \le k \le n-1, \\ \sigma_i\sigma_j = \sigma_j\sigma_i, & |i-j| \ge 2 \end{array} \right\rangle$$

and the discrete Heisenberg group

$$H(\mathbb{Z}) = \langle a, b \mid [[a,b],a] = 1,\ [[a,b],b] = 1 \rangle.$$

In the braid groups with the standard Artin generators, geodesic words correspond to minimal crossing number braid diagrams. I explore the classes of so-called homogeneous braids, such as positive or alternating braids. I proved that the latter form a right-angled Artin monoid. Using this, I gave a lower bound on the growth function Γ_n of the braid groups B_n

$$\log 4 \le \lim_{m\to\infty} \frac{\log \Gamma_n(m)}{m} \le \log 7.$$

It seems that all monoids of homogeneous braids are Artin monoids. Finally, I presented some sufficient general conditions on words to be geodesic. All this is a part of my attack of J. Stallings conjecture, which states that all geodesic words in the braid groups are expandable in a certain way. I proved it for periodic braids and made an advance toward reducible braids, but the pseudo-Anosov case is still open. In the discrete Heisenberg group, words in the standard two generating set correspond to polygonal chains on the plane lattice. Geodesic words are known to be simple polygonal chains. I found a beautiful connection between geodesic words and minimal perimeter polyomino. Namely, I showed that any geodesic word is a prefix of a dead-end word. It turns out that dead-end words correspond to closed simple polygonal chains that bound minimal perimeter polyominos. It would be interesting to find descriptions of geodesic words for other free nilpotent groups of rank two. My research interests include group theory and low dimensional topology. In my thesis, I would like to study connections between mapping class groups, 3-manifolds, and hyperbolic geometry.

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Raphael Appenzeller
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Ph.D. advisor: Marc Burger

Generalized buildings arising from symmetric spaces over non-Archimedean real closed fields

Real closed fields are ordered fields that generalize the real numbers and can be used as a base field when doing geometry. Symmetric spaces can be defined for non-Archimedean real closed fields $\mathbb{F}$. For instance, the upper half-plane model of the hyperbolic plane can be constructed as $\mathbb{F} \times \mathbb{F}_{>0}$. After identifying nearby points, these symmetric spaces are Λ-metric spaces, i.e. distances take values in an ordered group Λ. For the hyperbolic plane, this Λ-metric space turns out to be a Λ-tree, where geodesics correspond to infinite paths in the tree. In symmetric spaces of higher rank, the resulting space is a so called a Λ-building in the sense of Bennett, where the maximal flats give rise to larger flat structures. The most important tools to analyze these spaces are the isometry groups, which are linear algebraic groups over $\mathbb{F}$. One of the important features of these spaces is that the real spectrum approach to Higher Teichmüller Theory leads to a wealth of interesting actions of surface groups on Lambda buildings with unexpected properness properties. I started thinking about these objects in my Bachelor and Master thesis. I recently started my PhD at ETH Zürich under the supervision of Marc Burger to continue this work. While it is possible to show many things when working with concrete matrix groups, it would be favorable to develop the theory of Lie groups and Lie algebras for real closed fields in general. This includes structure theory results such as the Cartan, Iwasawa and Bruhat decompositions.

Macarena Arenas
McGill University
Ph.D. advisor: Dani Wise

Hyperbolicity and cubical small-cancellation theory

A *cubical presentation* $X^* = \langle X | \{Y_i\} \rangle$ consists of a connected nonpositively curved cube complex X together with a collection of local isometries of connected non-positively curved cube complexes $Y_i \to X$, which we think of as "relator". The fundamental group of a cubical presentation is given by $\pi_1 X / \ll \{\pi_1 Y_i\} \gg$. This constitutes a generalisation of classical small-cancellation theory, because a standard presentation $\langle s_1, ..., s_t | R_1, ..., R_k \rangle$ can be interpreted as a cubical presentation by letting X be a bouquet of t circles and letting each Y be an immersed cycle.

In this context, a *piece* is the overlaps of a relator with another relator or with a hyperplane carrier. The $C(n)$ *small-cancellation condition* states that no essential closed path σ is the concatenation of fewer than n pieces, and the $C'(\frac{1}{n})$ *condition* states that if μ is a piece in an essential closed path σ, then $|\mu| < \frac{1}{n}|\sigma|$, where $|\mu|$ is the distance between endpoints of $\widetilde{\mu} \subset \widetilde{X}$.

When $\pi_1(X)$ is hyperbolic, it is known that the $C'(\frac{1}{14})$ condition on a presentation implies hyperbolicity of $\pi_1(X^*)$. I am working on generalising this to certain $C(13)$ presentations. It is also known that hyperbolicity of $\pi_1(X)$ implies cocompactness of the CAT(0) cube complexes dual to the wallspace associated to X^*, and I am working on understanding under what circumstances cocompactness holds in general.

While working with my advisor on these problems, we proved a linear isoperimetric inequality for genus-0 diagrams in hyperbolic groups, generalising the linear isoperimetric inequalities for disc diagrams and annular diagrams known to hold in the hyperbolic setting.

Shivam Arora
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Ph.D. advisor: Eduardo Martínez-Pedroza

Hyperbolic groups

The area I am currently interested in is Geometric group theory. The focus of the Geometric group theory is to use various techniques to unravel the relations between its algebraic properties and inherent geometry. The inherent geometric structure for finitely generated groups is unique up to quasi-isometry, and hence it is useful to study the properties that are preserved under quasi-isometric maps. One such property of metric spaces is being Gromov-hyperbolic. This property is a quasi-isometric invariant for geodesic metric spaces. The study of hyperbolic spaces gained prominent presence with the work of Gromov and has been studied by various mathematicians in the last couple of decades.

Below in two sections, a summary and context for the two of the projects we have worked on are given. In the end, the future and past work is mentioned.

Subgroups of discrete hyperbolic groups

A major theme in the understanding of group structure is understanding its subgroups, in particular, one of the natural problem for Hyperbolic groups is the following:

Problem. *Let G be a hyperbolic group and let $H \subseteq G$ be a finitely presented subgroup. Is H hyperbolic?*

The problem was settled in negative by the construction of famous counter-example by Bestvina-Brady in [1]. A result of S. M. Gersten answered the problem in positive for a class of groups. The results state

Theorem (Gersten [5])**.** *Let G be a hyperbolic group such that $\mathsf{cd}_{\mathbb{Z}}(G) = 2$. If H is a finitely presented subgroup, then H is hyperbolic.*

The cohomological dimension $\mathsf{cd}_R(G)$ of a group G with respect to a ring R is less than or equal to n if the trivial RG-module R has a projective resolution of length n.

The main motivation of our project was to investigate the hyperbolic groups with $\mathsf{cd}_R(G) = 2$ for $R = \mathbb{Q}$, the rational numbers. We wanted to generalize Gersten's result to the rational case because of the existence of hyperbolic groups of integral cohomological dimension three and rational cohomological dimension two. The nature of finitely presented subgroups of groups in this class was not known. The first examples of such groups were discovered by Bestvina and Mess [3] based on methods given by Davis and Januszkiewicz [4]. The class also contains finite index subgroups of hyperbolic Coxeter groups, examples that were discovered by Dranishnikov [12].

The result we obtained is the following:

Theorem (Arora, Martínez [6]). *Let G be a hyperbolic group such that $cd_{\mathbb{Q}}(G) \leq 2$. If H is a finitely presented subgroup, then H is hyperbolic.*

Application

The results enlarge the class of the groups for which the above-mentioned problem has an affirmative answer. In particular, it includes groups that admit torsion, specifically, to the class of hyperbolic groups G admitting a 2-dimensional classifying space for proper actions $\underline{E}G$. The minimal dimension of a model for $\underline{E}G$ is denoted by $\underline{\mathsf{gd}}(G)$.

Corollary (Arora, Martínez [6]). *If G is a hyperbolic group such that $\underline{\mathsf{gd}}(G) \leq 2$, then any finitely presented subgroup is hyperbolic.*

Hyperbolic topological groups

A compactly generated locally compact (LC-) group admits a natural word metric that is unique up to quasi-isometry. Moreover we can associate geodesic metric spaces quasi-isometric to this metric space along with an action of G given by the following theorem.

Theorem (Prop 2.1 [10]). *Let G be a compactly generated, locally compact group. There exists a finite-dimensional (in the sense of topological dimension) locally compact geodesic metric space X with a continuous, proper, cocompact G-action by isometries.*

A compactly generated locally compact group is said to be hyperbolic if the geodesic space described in the above theorem is Gromov hyperbolic. Since such a space is unique up to quasi-isometry and hyperbolicty is preserved under quasi-isometry, it is a well-defined property of LC-groups.

In case the LC-group is totally disconnected (TD), the geodesic space is a locally finite graph.

Theorem (Theorem 2.2 [9]). *A TDLC-group G is compactly generated if and only if it acts vertex transitively with compact open vertex stabilizers on a locally finite connected graph Γ called Cayley-Abels Graph.*

A lot of recent work in the study of totally disconnected locally compact(TDLC) groups employed geometric group theory techniques to study the properties of the groups. Some of them are [9], [10], [11]. hyperbolic TDLC-groups have been studied in [10]. In this project, we gave a characterization of hyperbolic TDLC-groups in terms of isoperimetric inequality.

Theorem (Arora-Castellano-Cook-Martínez[7]). *A compactly generated TDLC-group G is hyperbolic if and only if G is compactly presented and satisfies the weak linear isoperimetric inequality.*

The definition of weak-isoperimetric functions is analogous to the discrete case, with modules considered in the category of rational discrete modules introduced in [8]. Let $\mathbb{Q}$ denote the field of rational numbers, and let ${}_{\mathbb{Q}G}\mathbf{mod}$ be the category of abstract left $\mathbb{Q}G$-modules and their homomorphisms. A left $\mathbb{Q}G$-module M is said to be *discrete* if the stabilizer

$$G_m = \{g \in G \mid g \cdot m = m\},$$

of each element $m \in M$ is an open subgroup of G. The full subcategory of ${}_{\mathbb{Q}G}\mathbf{mod}$ whose objects are discrete $\mathbb{Q}G$-modules is denoted by ${}_{\mathbb{Q}G}\mathbf{dis}$. It was shown that ${}_{\mathbb{Q}G}\mathbf{dis}$ is an abelian category with enough injectives and projectives. Hence cohomological dimension and finiteness properties of a TDLC group can be defined analogously and is a generalization of discrete case.

We studied the groups with cohomological dimension 2 and generalized our previous result for discrete groups.

Theorem (Arora-Castellano-Cook-Martínez[7]). *Let G be a hyperbolic TDLC-group with $\mathsf{cd}_{\mathbb{Q}}(G) \leq 2$. Every compactly presented closed subgroup H of G is hyperbolic.*

Future problems

In the future, I am planning to study more large-scale properties of totally disconnected locally compact groups using geometric techniques.

Past work

In past I have studied group theory. In my Master's project I studied z-classes of groups. It is a equivalence relation on groups defined as, two elements being equivalent if their centralizers are conjugate. It was introduced by Ravi Kulkarni in [13] and has interesting connections with the dynamical properties of the group. There was an open question in [14] related to the characterization of p-groups with maximum number of z-classes. We solved the problem for a specific case in [15].

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Cubulating Groups

My current research is on cubulating groups. The topic of groups acting on CAT(0) cube complexes is an active research area - proving a group is virtually special is often an approachable method to prove that a group is linear and residually finite [1]. Indeed, many groups are known to be virtually special, such as small cancellation groups [2], and "most" groups are cubulable [3]. Recent work (see [4]) has meant that proving a group is virtually special is now far more accessible, and has already been used to prove that random groups with relator length 4 are virtually special [5]. I am interested in applying this to other models of random groups.

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Rigidity of k-separating curve graphs

The curve graph $\mathcal{C}(S)$ associated to a surface S encodes in a geometric and combinatorial structure some information about the curves on the surface. It has a vertex for every curve on S and an edge between two curves if they admit disjoint representatives. The extended mapping class group of a surface acts naturally on the curve graph and, apart from sporadic cases, the action is faithful. *A priori*, though, the automorphisms of the curve graph are purely combinatorial, so it is the following result, originally due to Ivanov [2], is striking.

Theorem ([4, Main Theorem]). *Apart from some sporadic cases, the natural map* $Mod^{\pm}(S) \longrightarrow Aut(\mathcal{C}(S))$ *is an isomorphism. We say that the curve complex is rigid.*

Similar rigidity results have been proven for variations of the curve complex, such as the separating curve graph, in which edges are still defined by disjointness, while vertices consist of separating curves only [3, Thm. 1.2]. It is possible to restrict the set of vertices even more by considering separating curves such that both subsurfaces obtained by cutting along the curve are complex enough: for curves not bounding any pair of pants the resulting complex has been proven to be rigid [1, Thm. 1.1]. Ivanov claimed that the most part of these complexes, apart from some sporadic cases, should be rigid: I am currently working on proving the following conjecture, which would strengthens a result by McLeay [5, Thm. 1.5], as well as trying to generalise the result for surfaces with genus.

Conjecture. *Let* $p \geq 2k+1$*. Then the subgraph of* $\mathcal{C}(S_{0,p})$ *spanned by curves which do not bound any subsurface less complex than a* $S_{0,k+1}$ *is rigid.*

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Benjamin Barrett
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Boundaries of hyperbolic groups

It is well known that a general finitely presented group can have very nasty properties, but many of these properties are avoided if the group is assumed to admit a nice action by isometries on a metric space with a negative curvature property, such as Gromov hyperbolicity. There is a sense in which a random group admits such an action, and this is also true of some groups of classical interest, such as fundamental groups of closed Riemannian manifolds with negative sectional curvature. Given a hyperbolic group G, one can define the *Gromov boundary* ∂G comparable to the spherical boundary of hyperbolic space $\mathbb{H}^n$. Certain connectivity properties of ∂G are equivalent to important algebraic properties of Γ. For example, it is a theorem of Stallings [5] that ∂G is disconnected if and only if G admits a splitting as an amalgamated free product or HNN extension over a finite subgroup. Similarly, by a theorem of Bowditch [4] splittings of G over virtually cyclic subgroups can be seen in the structure of local cut points in ∂G. Motivated by this, I am interested in the link between the geometry of large balls in the Cayley graph of a hyperbolic group G and the topology of the Gromov boundary ∂G. By identifying properties of such balls with interesting topological features of ∂G one can sometimes show the question of the existence of such features to be algorithmically decidable. The main result of my PhD [1] is the detectibility of cut pairs in ∂G, which implies the computability of Bowditch's canonical JSJ decomposition for G. More recently I have been interested in the detection of Čech cohomology and local contractibility of hyperbolic group boundaries. Some early work in this direction can be found in my thesis [2], which expands on [3].

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Generalising Mirzakhani's curve counting

I am in the 2nd year of my PhD. The mapping class group of a surface S, denoted by $\mathrm{Mod}(S)$, is the group of orientation-preserving homeomorphisms of S up to homotopy. $\mathrm{Mod}(S)$ acts naturally on homotopy classes of closed curves on S. If S is given a hyperbolic metric, then each homotopy class has a unique geodesic representative, and thus a well-defined length. My goal is to generalise Mirzakhani's theorem [1] regarding counting curves on surfaces. Let S have genus g and n boundary components. Then Mirzakhani proved that the number of curves in each mapping class group orbit of length at most L is asymptotic to a constant times $L^{6g-6+2n}$. I have proved that the theorem is true when curves are replaced by arcs between boundary components, if S has non-empty boundary. Currently, I am working to determine to what extent the theorem holds when $\mathrm{Mod}(S)$ is replaced by a subgroup. Erlandsson-Parlier-Souto ([2],[3]) have also proved Mirzakhani's result for more general notions of length, using geodesic currents.

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Average Dehn function of finitely generated groups and LG-rigidity of transitive graphs

I am 1st year PhD student under the supervision of Romain Tessera. I am mainly interested in two subjects.

The Dehn function of a finitely presented group G is a geometric invariant which corresponds to the complexity of solving the word problem in G.
Given a presentation of G, we can associate to it a a simply connected 2-complex X. The *Dehn function* $\delta_G(n)$ corresponds to the maximal area of a loop of length n in X. The *averaged Dehn function* corresponds then to the average area (w.r.t a random walk on G) of a loop of length n. Following the work [1] of R.Young where he studies nilpotent groups, we investigate the averaged Dehn function of some groups of exponential growth.

LG-rigidity of graphs : A vertex-transitive graph X is said to be *LG-rigid* (for local-to-global rigid) at scale R if every graph Y which have the same balls of radius R as X, is covered by X. Following the work [2] of de la Salle and Tessera, we investigate which Cayley graphs and vertex transitive graphs have this property.

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Ph.D. advisor: Beatrice Pozzetti

Maximal representations in higher Teichmuller theory

I am writing my master thesis with Maria Beatrice Pozzetti: I am trying to rewrite the Bajaman inequality in the indefinite special orthogonal group of signature $(2, n)$. Maximal representations in Higher Teichmuller theory are the main topic. Other fields of interest, which I have also studied in the last years, are geometric group theory, Teichmuller theory, symmetric spaces, non-positively curved manifold (with a particular accent on the group actions).

Leonardo Biz
TU Dresden
Ph.D. advisor: Andreas Thom and Dr. Vadim Alekseev

Properties of the coarse boundary groupoid

Given a sequence $\{X_i\}_{i\in\mathbb{N}}$ of bounded degree finite graphs. In a coarse geometry point of view, I am interested in study which/how properties from this kind of sequences pass to correspondence properties on the "limit" and vice-versa. For the "limit" object, I mean, take the space of graphs $X\colon = \sqcup_i X_i$ with a compatible metric d, we can define the (coarse) translation groupoid that captures the coarse information associated to X and restrict it to the Stone-Cech compatification of X. More precisely, define, for all $R > 0$, the metric coarse geometric structure generated by the collection of $E_r = \{(x, y) \in X \times X;\ d(x, y) < R\}$. Now, we can define $G(X) = \cup_{R>0}\overline{E_R}$, where $\overline{E_r}$ is the Stone-Cech compatification in $\beta X \times \beta X$. With the pair groupoid operation $G(X)$ is a groupoid (see [2]). In particular, it is saturated in $\partial\beta X\colon = \beta X - X$, so we can restrict it to $\partial G(X)\colon = G(X)|_{\partial\beta X}$ and call it by *boundary groupoid*. This groupoid has a description in terms of ultralimits and this gives us this "limit" notion. With this machine, some results known for the relation between box spaces and the group were partially generalized for sofic approximation and the group.

Theorem ([1]). *Let Γ be a finitely generated group, $\mathcal{G}$ a sofic approximation of the group and X be the space of graphs constructed from the sofic approximation.*

(1). If X has Property A then Γ is amenable.

(2). If X is asymptotically embeddable into a Hilbert space then Γ is a-T-menable.

(3). If X had boundary geometric property (T) then Γ has property (T).

I am interested to know when the converse of this theorem holds. Some progress were made about the converse of the first two items when we replace Γ by $\partial G(X)$.

Theorem. *Let $\mathcal{X} = \{X_i\}_{i\in\mathbb{N}}$ be a sequence of finite bounded degree graphs and $R_{\partial G(X)}$ the equivalence relation induced by the boundary groupoid. The following statements are equivalent:*

(i). $(R_{\partial G(X)}, \hat{\mu}_\omega)$ *is amenable for all* $\omega \in \partial\beta\mathbb{N}$*;*

(ii). $\mathcal{X}$ *has property A on average along all* $\omega \in \partial\beta\mathbb{N}$*;*

(iii). $\mathcal{X}$ *is hyperfinite;*

(iv). $(R_{\partial G(X)}, \hat{\mu}_\omega)$ *is hyperfinite for all* $\omega \in \partial\beta\mathbb{N}$*.*

The converse of the second question is a work in progress.

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Curvature tests and hyperbolicity

In my licentiate thesis I began studying Whitehead's asphericity conjecture [7]. It states that every subcomplex of a 2-dimensional aspherical CW-complex is itself aspherical. One of the main motivations behind Whiteheads question was the asphericity of knot complements, which was proved in 1957 by Papakyriakopoulos. A generalization of knot complements are LOT complexes (labeled oriented trees) [3]. These are combinatorial 2-complexes that arise as the complex associated to a group presentation which can be defined in terms of a labeled oriented tree. Howie proved that if the Andrews-Curtis Conjecture were true, then the asphericity of LOT complexes would imply the Whitehead conjecture for the finite case [4]. There are many ways in which one can prove the asphericity of numerous families of LOTs. One of them is via curvature tests, such as Gersten's weight test [2]. Cells in a combinatorial 2-complex can be thought of as polygons. It is possible to assign numbers (*weights*) to the angles of these polygons. These angles are in correspondence with the edges of the links of the combinatorial 2-complex. Comparing this *weight function* to the angles in a polygon in traditional non-positively curved spaces, and drawing a parallel with Cartan-Hadamard's theorem, Gersten found a test for combinatorial 2-complexes that implied asphericity. If instead of assigning angles in a non-positively curved fashion, one does so with negative curvature in mind, the same techniques imply hyperbolicity. More specifically, a linear isoperimetric inequality can be found. After Gersten's original result, similar arguments and ideas appeared in the literature [5,8]. By giving a unified version of these curvature tests in my thesis, together with Minian we were able to see that one of Wise's questions concerning the hyperbolicity of complexes with negative sectional curvature, could be answered positively [8]. Moreover, complexes with negative planar curvature are hyperbolic. In 2019 I started my PhD under the supervision of Elías Gabriel Minian with the objective of continuing the study of nonpositivey curved groups and spaces and their relation with curvature tests. In [1] we introduced the notion of strictly systolic angled complexes, which generalize Januszkiewickz and Świątkowski's 7-systolic simplicial complexes [6]. Similarly, one could define systolic angled complexes as a generalization of their 6-systolic simplicial complexes. Strictly systolic angled complexes are hyperbolic, and we used them to prove hyperbolicity of a wide family of one-relator groups that generalizes classical small-cancellation hypothesis such as $C'(1/6)$ and $C'(1/4) - T(4)$. A future line of work would be to analyze these complexes in more detail and see if the advances that have been made in the simplicial non-positive curvature case can be obtained in this new context. I am also interested in finding other families of groups for which appropriate strictly systolic complexes can be constructed in order to prove hyperbolicity.

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Some groups of type FP

Let X be an acyclic 2-complex such that the attaching map of each 2-cell is an embedding in the 1-skeleton, the set of 2-cells satisfy $C'(\frac{1}{6})$ and the girth of the 1-skeleton is greater than 12 and less than half the boundary length of the smallest 2-cell. To such a complex we associate a continuous family of infinitely presented groups that are indexed by subsets of $\mathbb{Z}$ and can be defined by a graph satisfying the graphical small cancellation condition $C'(\frac{1}{6})$. Each of these groups embeds in a group of type FP and, when the subset of $\mathbb{Z}$ is infinite, the group is not finitely presented. The only previous such constructions, see [1] and [2], rely on Morse Theory and CAT(0) cube complexes. The construction here uses neither. Current work includes solving the word problem for families of these groups and showing that each such family contains continuously many quasi-isometry classes.

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Acute triangulations of polyhedra

Interest in acute triangulations dates back to Aristotle's tetrahedral trip-up, when he falsely claimed that the regular tetrahedron *did* tesselate 3-space (this mistake went uncorrected for almost 2000 years). In more recent years, acute triangulations have surfaced in a wide range of areas of mathematics, to name a few, they play an improtant role in computer graphics, applied and computational mathematics where a "nice" triangulation is required for finite element methods and various other tools, mathematical biology, network modelling, or even space-time meshing in physics. Perhaps more importantly, they have proven to be a beautiful and difficult geometrical puzzle of low-dimensional geometry and topology. Indeed, because of the combinatorial obstructions given by the angle constraints (and a direct application of the Dehn-Sommerville equations), they cannot exist in dimensions greater than 4. Even in dimension 4, there is no acute triangulation of the tesseract, no periodic acute triangulation of $\mathbb{R}^4$, and no acute triangulation in which the angles are bounded away from $\frac{\pi}{2}$ (it is conjectured there is no acute triangulation *at all*). While the problem of acute triangulations in the plane has been extensively studied and the existence of an acute triangulation proven for any plane polygon, the study of the 3 dimensional case is very recent and ongoing. Surprisingly perhaps, the only known acute triangulations of polyhedra were given by Przytycki, Pak and Kopzcynski [1] using a very beautiful construction using the stereographical projection of the 600-cell. The three authors conjectured that any polyhedron should admit an acute triangulation. However, adapting their construction to obtain acute triangulations of *any* tetrahedron has proven to be quite challenging. In more recent work, I have become very interested in ties between acute triangulations with hyperbolic geometry. A very interesting and closely related result is due to Walsh and Kim [2], who showed the following: if T is an abstract triangulation of $\mathbb{S}^2$, then T can be realised as an acute triangulation if and only if the right-angled Coxeter group associated to T is isomorphic to a hyperbolic right-angled reflection group.

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Separability within the class of alternating groups

What can we infer about a group from its finite quotients? What if we only look at quotients in certain class of groups? When do two elements remain distinct in one of these quotients? When does an element remain disjoint from a subgroup after mapping to one of the groups? When do two non-conjugate elements remain non-conjugate in some quotient? These are the questions I care about. For example in hyperbolic orientable surface groups, given an infinite index finitely generated subgroup and a collection of elements not in this subgroup, we can find a map onto an alternating group, such that none of these elements is sent to the image of the subgroup.

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Boundaries of Hyperbolic Spaces

I am second year Master's student at New York University. I am interested in using geometric and topological methods to study hyperbolic groups. Let Γ be the Cayley graph of a group G with respect to a set S of generators. We can endow Γ with a word metric, making it a length space. The group G is said to be hyperbolic if the space Γ is Gromov-hyperbolic-i.e., if Γ satisfies the thin triangle condition. The Gromov-hyperbolicity of G is independent of S, and it is an invariant under quasi-isometry. The Gromov boundary of a hyperbolic space is, loosely speaking, the set of all points at infinity. Topological ideas about the boundaries of hyperbolic groups were generalized to relatively hyperbolic groups by Bowditch in 1999. A group G is said to be hyperbolic relative to a "peripheral subgroup" H if the coned off Cayley graph $\hat{\Gamma}(G, H)$ is (1) Gromov-hyperbolic and (2) fine: for each integer L, every edge belongs to only finitely many simple cycles of length L. Now we can associate to such a group (Γ, G) a canonical boundary $\delta(\Gamma, G)$, which is a compact metrisable space on which Γ acts by homeomorphism. I am interested in investigating the boundary to study further topological properties of the underlying group G and its peripheral subgroups.

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Matthew Conder
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Discrete and free two-generated subgroups of SL_2

Two-generated subgroups of $\mathrm{SL}_2(\mathbb{R})$ have been widely studied in the literature. In particular, it is well known that the Ping Pong Lemma, applied to the action of $\mathrm{SL}_2(\mathbb{R})$ by Möbius transformations on the hyperbolic plane, can be used to determine whether or not many of these two-generated subgroups are either discrete (with respect to the topology inherited from $\mathbb{R}^4$) and/or free of rank two. Moreover, there is a practical algorithm which, given any two elements of $\mathrm{SL}_2(\mathbb{R})$, will determine after finitely many steps whether or not they generate a subgroup which is both discrete and free of rank two; see [2]. On the other hand, two-generated subgroups of SL_2 over other fields (in particular, other locally compact infinite fields) are not as well studied. My research involves looking at the case of $\mathrm{SL}_2(K)$, where K is a non-archimedean local field (for instance, the p-adic numbers $\mathbb{Q}_p$). Such groups act by isometries on the corresponding Bruhat-Tits tree, and studying this action leads to an algorithm which determines after finitely many steps whether or not any given two-generated subgroup of $\mathrm{SL}_2(K)$ is both discrete (with respect to the topology inherited from K^4) and free of rank two; see [1]. The basis of this algorithm involves comparing the translation lengths of certain elements, and this is general enough so that the algorithm can be applied to some other classes of groups which act by isometries on a locally finite tree. I am hopeful that this algorithm might have some other applications too, such as using a similar method to determine whether or not any given n-generated subgroup of $\mathrm{SL}_2(K)$ is discrete and free of rank n.

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Arman Darbinyan
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Groups, computability, and computational complexity

My research is mostly about computable aspects of groups. I am especially interested in connections between logic, computability theory, computational complexity and algebraic/ geometric/ topological properties of groups. My previous research projects were mostly concerned with the word and conjugacy problems in various settings. My current projects are also about formal languages, computational complexity, computable orders on groups, simple groups, and group embeddings. I also have developing interest in random walks on groups and dynamics.

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Lucas H. R. de Souza
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Research in perspective compactifications

I work with perspective group compactifications of groups. Let G be a group acting by homeomorphisms on a Hausdorff compact space Y and properly discontinuously and cocompactly on a Hausdorff locally compact space X. Let $Z = X\dot{\cup}Y$ be a Hausdorff compact space (with canonical uniform structure $\mathcal{U}$) such that extends the topologies of X and Y in such a way that the induced action of G in Z is by homeomorphisms. We say that Z is perspective if for every $u \in \mathcal{U}$ and every compact $K \subseteq X$, the set $\{g \in G : gK \notin u\}$ is finite. This means that whenever K gets closer to the boundary of X (by elements of G) it becomes arbitrarily small. In the case when $G = X$, with the discrete topology, there is another equivalent definition of perspectivity: the right multiplication action on G extends continuously to the identity on Y. Some examples of perspectivity are:

1. If Z has the convergence property, then Z has the perspective property [3].
2. If $(\tilde{X}, Y)$ is an E$\mathcal{Z}$-structure on a group G, then $\tilde{X}$ has the perspectivity property with respect to the action of G on it.
3. If G acts by isometries on a proper CAT(0) space X, then the visual compactification of X is perspective. In particular, the compactification of $\mathbb{Z}^n$, for $n \geqslant 2$, with the visual boundary of $\mathbb{R}^n$ is perspective but it does not have the convergence property since the action on the boundary is trivial.
4. The Martin compactification of every finitely generated relatively hyperbolic group with virtually abelian parabolic subgroups is perspective (but does not have the convergence property unless all parabolic subgroups are virtually cyclic) [2].
5. Gerasimov and Potyagailo showed in [4] that the pullback problem for convergence actions does not have a solution in general. A pullback may not exist even when both actions

are relatively hyperbolic and the group is not finitely generated. However, that is not the case for perspective property. So families of convergence actions that do not have pullback convergence actions give rises to spaces that do have the perspective property but its associate actions must not have the convergence property.

Let's consider the category $Comp(G)$ (respec. $Comp(\varphi)$, where $\varphi : G \curvearrowright X$ is a properly discontinuous cocompact action), whose objects are compact spaces of the form $G\dot{\cup}Y$ (respec. $X\dot{\cup}Y$) such that the left multiplication action (respec. φ) extends continuously to the whole space and the morphisms are equivariant continuous maps which restrict to the identity in G (respec. identity in X). Let's consider the $Pers(G)$ (respec. $Pers(\varphi)$), the full subcategory of $Comp(G)$ (respec. $Comp(\varphi)$) where the objects are perspective compactifications. If $K \subseteq X$ is a compact subspace such that $\varphi(G, K) = X$), then we are able to define two functors that preserves boundaries $\Pi_K : Comp(G) \to Comp(\varphi)$ and $\Lambda_K : Comp(\varphi) \to Comp(G)$. However such functors depend of the choice of K, send some Hausdorff spaces to non Hausdorff spaces and do not form equivalences of categories. If we restrict those functors to perspective compactifications, we have two functors $\Pi : Pers(G) \to Pers(\varphi)$ and $\Lambda : Pers(\varphi) \to Pers(G)$ that do not depend of the choice of K and are inverses of each other. We proved the following theorem:

Theorem ([5])**.** *: Let G be a group, X a Hausdorff locally compact space and $\varphi : G \curvearrowright X$ a properly discontinuous cocompact action. Then, the categories $Pers(G)$ and $Pers(\varphi)$ are isomorphic. So there is a correspondence between those classes of compact spaces. Moreover, the functor from Π sends objects of the form $Z = G\dot{\cup}Y$ to objects of the form $Z' = X\dot{\cup}Y$.*

In another words: we are able to transfer the boundaries of perspective compactifications from the group to X and vice versa.

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Big mapping class groups

I am most interested in mapping class groups of infinite type surfaces or big mapping class groups [1]. In particular I like to think about which properties of the infinite symmetric group translate over to these more complicated groups. My current results involve the mapping class group of the surface with infinite genus and one end. I am trying to determine whether the mapping class group of this surface is strongly bounded which means any action by isometries on a metric space has bounded orbits. Another question I have thought about is whether these mapping class groups have automatic continuity, i.e. any homomorphism to a Polish group is continuous. My other interests include Culler-Vogtmann Outer space, Bestvina-Bromberg-Fujiwara projection complexes, and their applications to mapping class groups [2]. I hope to use projection complexes as a tool to study big mapping class groups and their stabilizer subgroups.

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Big mapping class groups and complexes

Currently, my main interests lie in the study of mapping class groups. I am particularly intrigued by all that we are able to learn about these groups by looking at their actions on suitably chosen graphs and complexes. My aim now is to dig deeper into the world of big mapping class groups, the mapping class groups of infinite-type surfaces, and search for new complexes and group actions which could show us analogous or perhaps different results as compared to the finite-type case. For instance: is there any hope of finding a nice dynamical description of big mapping classes in the vein of the Nielsen-Thurston classification? How can we extend familiar group actions from the finite-type setting to big mapping class groups, and what objects can we come up with that are uniquely suited to the infinite-type surfaces? And how do we leverage the classification of noncompact surfaces of Kerekjarto/Richards in all of this?

I am also interested in exploring connections between big mapping class groups and other branches of low-dimensional topology. Really, I'm still learning and would be glad to chat with anyone about this or any related topics.

George Domat
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Big mapping class groups

The mapping class group, MCG(S), of a surface, S, is the group of homotopy classes of orientation preserving homeomorphisms of S to itself. When S is finite type, namely has finite genus and number of punctures, this group has been widely studied for the past several decades. Recently there has been a shift to also study these groups in the infinite type setting (infinite genus and/or infinitely many punctures). There has been much interest in which types of results from the finite type setting still hold in the infinite type setting and which do not. Most recently I have been interested in the first cohomology and homology of these groups. In [1] the authors computed the first cohomology of the pure mapping class group, the subgroup of MCG(S) which fixes the punctures or set of ends, when S has genus at least 2. In [2] we show that the first cohomology is trivial when S has one genus and build new cocycles when the genus is zero. It is a classic result of Ivanov in the finite type setting that the pure mapping class group has trivial abelianization once S has genus at least 3. The results on the first cohomology show that this is not always true in the infinite type setting. One can pass to a further subgroup, the closure of the compactly supported mapping classes, which is known to have trivial first cohomology and ask whether it has trivial abelianization. In this setting I hope to make use of the projection complex machinery of [3] to show that even in these subgroups we still do not have trivial abelianizations.

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Linearity of graph manifold groups

A group Γ is said to be *linear* if there is a faithful representation of Γ on some finite-dimensional vector space. If $\Gamma = \pi_1(M)$, where M is a connected manifold, we say that M is *linear*. Thurston asked if any closed, connected 3-manifold M is linear. To answer this question, it suffices to consider the case where M is orientable and irreducible. In this context, either M is geometric, in which case it is easily seen to be linear, or M can be decomposed along tori into geometric pieces, each of which is either hyperbolic or Seifert fibered. If at least one of these pieces is hyperbolic, Przytycki and Wise have shown that M must be linear. Otherwise, M is called a *graph manifold*. If such M is nonpositively curved (NPC), then M is linear by the work of Liu. Thus, to completely answer Thurston's question, it now suffices to prove or disprove linearity of non-NPC graph manifolds. I am now considering the special case where M is the mapping torus of a composition of iterates of right Dehn twists along disjoint essential simple closed geodesics on a closed oriented hyperbolic surface.

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Geometric group theory and computability

I am interested in connections between computability theory and algebraic/geometric properties of groups. My current projects concerns effective aspects of amenability, the topic initiated by Matteo Cavaleri, Adam R. Day, Tullio Ceccherini-Silberstein and some other researchers. One of the main results of my research states that when a group is computable, non-amenability is equivalent to effective paradoxical decomposability (i.e. all members of the decomposition are computable). This is based on some work in effective graph theory. We together with Aleksander Ivanov have found an example of a finitely presented group with decidable word problem where the problem if a finite subset generates an amenable subgroup is undecidable. The paper which contains this work is available at arXiv.

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Groups actions on Banach spaces

I am interested in Kazhdan type rigidity properties for group actions on Banach spaces. For my masters thesis, I have studied two generalizations of Kazhdan's Property (T) to the Banach space setting and their relations: Property (T_X), which is due to Bader, Furman, Gelander and Monod, and Property (F_X) (every action by isometries on the Banach space X has a fixed point), which generalizes Serre's Property (FH). For my PhD, I aim to further study the representation theory of groups in the setting of Banach spaces, Kazhdan type rigidity Properties (including Lafforgue's strong Banach property (T)) and applications to approximation properties of operator algebras.

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Local-to-global rigidity of transitive graphs

I'm working under the supervision of Romain Tessera on the LG-rigidity of graphs. Informally the question guiding my research is: for a transitive graph X, is every graph having the same local geometry as X covered by X? Formally, for some $R > 0$ and two transitive graphs X and Y, we say that Y is *R-locally X* if all its balls of radius R are isometric to ones in X. Now, we say that X is *local to global rigid* (for short *LG-rigid*) at scale R, if every graph which is R-locally X, is covered by X. This idea was first introduced by Benjamini and Ellis [1], who showed that the standard Cayley graph of $\mathbb{Z}^d$ is LG-rigid. Then, de la Salle and Tessera introduced a generalized definition and showed that Cayley graphs of groups with polynomial growth and no torsion element are LG-rigid [2]. Later they extented the result of LG-rigidity to Bruhat-Tits building of $SL_d(\mathbb{Q}_p)$ [3]. My research try to extend these rigidity results to other transitive graphs.

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Anarchy in the Hanna Neumann conjecture

Let F be a free group and $H, K \subseteq F$ subgroups. A theorem of Howson [2] states that if H and K are finitely generated then $H \cap K$ is a finitely generated free group. We denote $\overline{r}(H) = rank(H) - 1$ to be the *reduced rank* of the subgroup H. As stated originally in 1957 [4] the Hanna Neumann conjecture predicts that:

$$\overline{r}(H \cap K) \leq \overline{r}(H)\overline{r}(K)$$

In 2011 Joel Friedman proved the conjecture [1], and amazingly, within a week Igor Mineyev independently released his own proof [3]. Friedman's proof consisted of constructing sheaves on digraphs to formulated a twisted cohomology theory. The punch line is that one can construct an exact sequence of sheaves over digraphs where the digraphs the sheaves are over are not necessarily surjective. This furthers Walter Neumann's translation of the conjecture [5] which takes a question about groups to a question about graphs. A free group, represented as a graph of groups, has trivial vertex and edge stabilizers. A natural question is to ask if there is a similar Hanna Neumann statement when the edge and vertex groups are finite but not necessarily trivial. My research involves leaving the land of graphs for the theory of categories to tackle this more general problem. I also hope to find other group theoretic questions in which sheaves can help answer.

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Ph.D. advisor: Jérémie Brieussel and Constantin Vernicos

Horocycle product of hyperbolique metric spaces

The general subject of my research is quasi-isometric classes of groups.

More specifically it focuses on the horocycle product of hyperbolic metric spaces. This is a subset of the direct product defined thanks to busemann functions (see [2]).

The goal of my Phd is to obtain a generalisation of Eskin-Fisher-Whyte works on the Diestel-Leader graphs and Sol geometry (see [3]).

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Spaces of actions on CAT(0) cube complexes

I am a first-year postdoc in Bonn, where my mentor is Ursula Hamenstädt. My main research interest at the moment are CAT(0) cube complexes. These form a large class of non-positively curved cell complexes, which is important in the study of many classical groups — for instance hyperbolic 3–manifold groups, Coxeter groups, right-angled Artin groups, free-by-cyclic groups and small cancellation groups.

In joint work with J. Beyrer, we have shown that, in many cases, group actions on CAT(0) cube complexes are *marked-length-spectrum rigid* [2,3]. This means that, given a group G acting on two CAT(0) cube complexes X and Y, there exists a G–equivariant isometry between X and Y if and only if every element of G has the same (combinatorial) translation length in X and Y.

As an application, one obtains a compactification of outer space for the space of untwisted outer automorphisms of a right-angled Artin group [1]. This generalises the classical length-function compactification of Culler–Vogtmann's outer space and can be viewed as an analogue of Thurston's compactification of Teichmüller space.

These results suggest that, for an arbitrary group G, it is interesting to consider the space of all G–actions on CAT(0) cube complexes and how these actions can degenerate into G–actions on more general spaces. This is one of the things I am working on at the moment.

Earlier in my PhD, I studied horofunction compactifications of median spaces [4] and used these to prove a superrigidity theorem for actions of irreducible lattices in products of locally compact groups [5].

Other topics that I am interested in are measured group theory, conformal dimension, Anosov representations, bounded cohomology.

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Ph.D. advisor: Marc Burger

Semialgebraic methods in geometric group theory

Recently, I started a PhD project at ETH Zurich under the supervision of Marc Burger. The general program is the following: Given a finitely generated group Γ and a semisimple real algebraic group G, one considers the character variety $\chi(\Gamma, G)$ which is the quotient by G-conjugation of the space $\mathrm{Hom}_{\mathrm{red}}(\Gamma, G)$ of reductive representations. By the Richardson-Slodowy theory $\chi(\Gamma, G)$ is homeomorphic to a closed real semialgebraic subset of some $\mathbb{R}^n$ and thus admits the real spectrum compactification $\widetilde{\chi}(\Gamma, G)$ in the sense of Coste and Roy. Points of $\widetilde{\chi}(\Gamma, G)$ are given by representations of Γ into $G(\mathbb{F})$ for various real closed fields $\mathbb{F}$. The idea is now to look at components $\mathcal{C} \subset \chi(\Gamma, G)$ that consist of representations with specific geometric properties and to investigate to which extent these survive in the real spectrum compactification $\widetilde{\mathcal{C}}$ of $\mathcal{C}$. Past research includes my master's thesis written under the supervision of Sebastian Hensel at the University of Bonn. In my thesis, I investigated the homology of finite covers of graphs as a representation of the group of deck transformations and certain subrepresentations. To answer this question I used tools from representation theory as well as low-dimensional topology. In a certain sense, I tried to establish a dictionary between the two. Of main interest were primitive elements in free groups and commutators of such.

Jonas Flechsig
University of Bielefeld
Ph.D. advisor: Kai-Uwe Bux

Artin groups acting on arc matching complexes

In my PhD project I study the connectivity of arc matching complexes. The connectivity of these complexes is useful to answer the following question: Does a suitable familiy of subgroups in an Artin group generate (or even better: n-generate) this group?
A *matching* in a finite graph is a set of disjoint edges. Ordered by inclusion matchings of a fixed graph define a finite simplicial complex.
Rather than considering this combinatorial situation I embed the vertex set of a graph in a surface - for example a disk. Instead of edges connecting vertices I look at arcs connecting marked points. Collections of homotopy classes of disjoint arcs now build an (infinite) simplicial complex - an *arc matching complex*.
With the help of discrete Morse theory one is able to show, that the degree of connectivity of arc matching complexes typically increases with the number of marked points in the surface.
Braid groups act in a natural way on these arc matching complexes. With the help of the connectivity results for the arc matching complexes, this action gives rise to study the higher generation of braid groups by a familiy of stabilizer subgroups.
I try to extend the techniques used in the surface case to the situation where one considers some easy 2-orbifolds instead of the surface.

Dominik Francoeur
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Groups acting on rooted trees

I mostly work with groups acting on rooted trees, and in particular with branch groups and groups defined by Mealy automata. Such groups are of special interest in group theory since they can have some very unusual properties that do not exist in more classical settings. In particular, one can find among them infinite finitely generated torsion groups, groups of intermediate growth and amenable but not elementary amenable groups. Therefore, they are a rich source of potential examples and counterexamples to various questions. I am very interested in studying the growth of groups acting on rooted trees, and in particular in recognising which groups are of intermediate growth. In [1], I improved a criterion originally developed by Bartholdi and Pochon to detect intermediate growth in a special class of branch groups, and I applied this criterion to new examples. On the other end of the spectrum, I proved in [2], together with Ivan Mitrofanov, that a group generated by an invertible and reversible Mealy automaton contains a free subsemigroup of rank two (and is thus of exponential growth) if and only if it contains an element of infinite order. In particular, no infinite group of polynomial growth can be found among those groups. I am also interested in the algebraic structure of groups acting on rooted trees. In a work with Alejandra Garrido, I proved in [3] that there exist branch groups of intermediate growth that admit maximal subgroups of infinite index. More generally, I would like to better understand the relationship between the properties of such groups and the properties of their actions on a rooted tree. I am also greatly interested in similar questions in other groups that do not necessarily act on trees in interesting ways, yet are still similar in some aspects, such as Thompson's groups or groups acting on Cantor spaces.

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Dynamical properties of groups

My main research interests fall into the intersection of group theory and dynamics. I'm generally interested in questions of the form "What do the (geometric, algebraic, structural) properties of the group have to do with what kinds of (Borel, Topological, Measurable) actions the group admits". Often this work will end up involving analyzing, or constructing, some sort of structures of a combinatorial nature on different groups. Some particular objects I'm very interested in studying include Borel equivalence relations, random walks on groups (and in particular the Poisson boundary), symbolic dynamics on groups, automorphism groups of subshifts, and entropy theory for both amenable and non-amenable groups. Some group properties I find especially interesting are amenability, the ICC (infinite conjugacy class groups) property, polynomial growth, solvable groups, and hyperbolic groups. I'm particular fond of classification results. Some particular projects that I've worked on include

- classifying which countable groups admit measures with non-trivial Poisson Boundaries (Joint with Yair Hartman, Omer Tamuz, and Pooya Vahidi Ferdowsi),
- classifying which countable groups admit proximal topological actions (Joint with Omer Tamuz and Pooya Vahidi Ferdowsi),
- enumerating the Borel Cardinilaties of countable normal subgroups (Joint with Forte Shinko).

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The first order theory of acylindrically hyperbolic groups

The class of acylindrically hyperbolic (a.h) groups gathers many well-studied groups under one roof, and the seemingly weak acylindricity condition has been proven to have potent consequences. It is therefore not surprising that this class of groups has been in the mainstream of geometric group theory since it was introduced by Osin in 2016; however, not much is known about the first order theory of a.h groups. In previous work, I proved a version of Merzlyakov's theorem (a group-theoretic variant of the implicit function theorem) for torsion-free a.h groups. Recently, in joint work with Simon André, the assumption regarding torsion has been removed, which resulted in the following generalized version of Merzlyakov's theorem, valid for all a.h groups:

Theorem. *Let G be an a.h group, let $\boldsymbol{y}$ and $\boldsymbol{x}$ be tuples of variables and let $\boldsymbol{a}$ be a tuple of elements from G. Suppose that $G \models \forall \boldsymbol{y} \exists \boldsymbol{x} \bigvee_{i=1}^{k} (\Sigma_i(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{a}) = 1 \wedge \Psi_i(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{a}) \neq 1)$ where each $\Sigma_i(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{a})$ and each $\Psi_i(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{a})$ is a finite collection of words in the letters $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{a}$. Denote by $K(G)$ the maximal normal finite subgroup of G and by $\mathrm{Inn}_G(K(G)) \leq \mathrm{Aut}(K(G))$ the subgroup of automorphisms of $K(G)$ which extend to inner automorphisms of G. Then for every tuple $\Phi = (\phi_1, ..., \phi_n)$ (of the same arity as $\boldsymbol{y}$) in $\mathrm{Inn}_G(K(G))$ there exist $1 \leq j \leq k$, a tuple $\boldsymbol{u}_\Phi$ of elements from G and an epimorphism r_Φ from $G_{\Sigma_j} = \langle \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{a} | \Sigma_j(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{a}) = 1 \rangle$ onto*

$$\left\langle \boldsymbol{y}, K(G) | \forall k \in K(G)\ y_i k y_i^{-1} = \phi_i(k) \right\rangle *_{K(G)} \langle \boldsymbol{a}, \boldsymbol{u}_\Phi \rangle$$

such that $r_\Phi(\boldsymbol{y}, \boldsymbol{a}) = (\boldsymbol{y}, \boldsymbol{a})$ and every component of $\Psi_j(r_\Phi(\boldsymbol{x}), \boldsymbol{y}, \boldsymbol{a})$ is non-trivial.

Using techniques from [1] and relying on the theorem above, we were able to conclude that the natural embedding $G \hookrightarrow G *_{K(G)}$ is an $\exists\forall\exists$-elementary embedding for every a.h group G. Another corollary of this theorem and [2, Theorem 6.3] is that all a.h groups have the same *positive* theory (which coincides with that of a non-abelian free group). This gives an alternative proof to the fact that a.h groups have infinite verbal width (proven by Bestvina, Bromberg and Fujiwara in 2018).

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Isoperimetry, Littlewood functions, and unitarisability of groups

Let us assume that Γ is a discrete group. A representation $\pi : \Gamma \to B(H)$, where H is a Hilbert space, is called uniformly bounded, if $\sup\limits_{g\in\Gamma} \|\pi(g)\| < \infty$. A representation $\pi : \Gamma \to B(H)$ is called unitarisable, if there exists an operator $S : H \to H$ such that $S^{-1}\pi(g)S$ is a unitary representation for any $g \in \Gamma$. A group Γ is called unitarisable if any uniformly bounded representation is unitarisable. It is known that amenable groups are unitarisable. It has been open ever since whether this is a characterization of unitarisability (this questionn is called the Dixmier's problem). The question remains open only for non-amenable groups without free subgroups.

One of the approaches to study unitarisability and amenability is related to the space of the Littlewood functions $T_1(\Gamma)$.The latter is the space of all functions $f\colon \Gamma \to \mathbb{C}$ admitting a decomposition

$$f(x^{-1}y) = f_1(x,y) + f_2(x,y) \qquad \forall x, y \in \Gamma$$

with $f_i\colon \Gamma \times \Gamma \to \mathbb{C}$ such that both of the following are finite:

$$\sup_x \sum_y |f_1(x,y)| \quad \text{and} \quad \sup_y \sum_x |f_2(x,y)|.$$

The connection is as follows. First, Γ is amenable if and only if $T_1(\Gamma) \subseteq \ell^1(\Gamma)$. Secondly, if Γ is unitarisable, then $T_1(\Gamma) \subseteq \ell^2(\Gamma)$. Thirdly, if Γ contains a non-abelian free subgroup, then $T_1(\Gamma) \nsubseteq \ell^p(\Gamma)$ for all $p < \infty$.

It turned out that we can say something more about non-amenable groups. More precisely, it is true that any non-amenable group Γ there exists $p > 1$ such that

$$T_1(\Gamma) \nsubseteq \ell^p(\Gamma).$$

This result inspired us to define the Littlewood exponent $\mathrm{Lit}(\Gamma) \in [0, \infty]$ of a group Γ as follows:

$$\mathrm{Lit}(\Gamma) = \inf\big\{p : T_1(\Gamma) \subseteq \ell^p(\Gamma)\big\}.$$

Moreover, there is a connection between the Littlewood exponent and the geometry of Γ. More precisely, $\mathrm{Lit}(\Gamma)$ is related to the behavior of a Cayley graph $Cay(\Gamma, S)$ when one increases generating sets. This connection allows us to prove that the invariant $\mathrm{Lit}(\Gamma)$ is not trivial (that is $\mathrm{Lit}(\Gamma) \notin \{0, 1, \infty\}$) and construct a group Γ with $1 < \mathrm{Lit}(\Gamma) < \infty$. It also allows us to estimate this invariant for some complicated groups (i.e. for Burnside groups of the large exponent) and find some geometric applications.

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Geometry in higher rank

Main interest

I work on discrete groups acting in semi-simple Lie groups. My principal objects of study are geometric and dynamical invariants: the critical exponent and the Hausdorff dimension of limit sets. Critical exponent is defined by the exponential growth rate of points in an orbit of the group. Let $\Gamma \subset G$ be a discrete group acting on $X = G/K$ the Riemannian symmetric space associated to G then it is defined by:

$$\delta(\Gamma) := \limsup_{R\to+\infty} \frac{1}{R} \log Card\{\gamma \in \Gamma \,|\, d(\gamma o, o) \leq R)\}.$$

The limit set is the set of accumulation points of an orbit of Γ in the boundary of X. I studied these invariants in different cases, coming from different types of geometry: for $G = SL_2(\mathbb{R}) \times SL_2(\mathbb{R})$, for $G = SO(p, q)$ for $G = SL_n(\mathbb{R})$. See [1,2,3,4]

Other interests

I have also been interested in homogeneous dynamics. I studied with T. Dang some topological mixing properties for Weyl flows (In $SL_n(\mathbb{R})$ it corresponds to flows of the form $t \to \exp(t\,\mathrm{diag}(\mathrm{a}_1, ..., \mathrm{a}_\mathrm{n}))$ for some diagonal matrices) in the space of Weyl chambers (G/M). This is the natural generalization in higher rank of the geodesic flow [5]. Finally I have been working on a problem at the interface between geometry and probability. The problem is the following: take M a compact Riemanian manifold, take 3 random points X_1, X_2, X_3 (for the Lebesgue measure) on M. For almost all pairs of points there exists a unique minimizing geodesic between them. Join X_i to X_j by this minimizing geodesic. It gives a random triangle. The question is: "What is the property of this triangle of being homotopically trivial". In a joint work with A. Yarmola we answered completely the problem for flat tori. In a more recent work I also considered the problem of looking at a sequence of random points joining one to the next one by the minimizing geodesic The question is to understand the behavior at infinity. I showed that for negatively curved manifolds this random process is transcient and converges almost surely in the Gromov boundary of the fundamental group.

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Big topological groups

I am currently interested in finding natural examples of "big" topological groups (typically non-locally compact). A first source of examples is given by the homeomorphism group of *scattered spaces* (topological spaces that contain no nonempty perfect subspace), for which the topology of pointwise convergence is compatible with the group structure. The first uncountable ordinal, endowed with the order topology, is a scattered space, and its homeomorphism group features interesting properties, such as amenability or Roelcke precompactness [1]. Another source of examples comes from uniform spaces. Many known examples (homeomorphism group of locally compact spaces, general linear group of Banach spaces, automorphism group of first-order structures, and so on) can be seen as particular instances of a general framework of automorphism groups of some uniform spaces endowed with a collection of bounded sets. One interest of this framework is to behave well with respect to completeness and metrisability. In particular, it yields a natural Polish topology on the automorphism group of Polish locally Roelcke-precompact groups [2].

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Totally disconnected groups

A locally compact group G is an extension

$$\{e\} \to G^0 \to G \to G/G^0 \to \{e\}$$

with G^0 being the connected component of the identity and G/G^0 a totally disconnected locally compact (tdlc) group. Locally compact connected groups, such as G^0, are well understood using "approximation by Lie groups" (see [1]). On the other hand, the study of the totally disconnected groups, such as G/G^0 is relatively new. It was in the early 90's when Willis established the basics for the structure theory of totally disconnected groups (see [2]). Main examples of tdlc groups include algebraic groups over totally disconnected local fields and groups acting on trees. Every tree automorphism induces a homeomorphism on the boundary of the tree. A tree almost-automorphism is a homeomorphism of the tree boundary, which locally looks like the one induced by a tree automorphism. I'm mostly interested in non-linear totally disconnected groups. In particular, groups of automorphisms of (products of) regular trees, and groups of almost-automorphisms of regular trees. Consider the space of all closed subgroups of a given group. A probability measure on this space is called an invariant random subgroup if it is invariant under conjugation. For example, a Dirac measure on a normal subgroup is an invariant random subgroup. Groups of trees almost-automorphisms attract attention as their are a unique example of non-discrete groups, having a very strong sense of simplicity. That is, they admit no non-trivial invariant random subgroups (see [3]). However, they do admit rich dynamics when acting on the boundary of the tree. For example, in contrast to tree automorphisms, elements may have a behaviour which is hyperbolic on some areas and elliptic on others. Recently, in a joint work with Waltraud Lederle, we characterized the dynamics behaviour of a tree almost-automorphism, and showed that this characterization solves the conjugacy problem for this group. For now, I try to use this rich dynamics to construct certain dense free subgroups in the group. I'm also interested in groups acting on products of trees, and in what senses they behave similarly to higher-rank Lie groups. In particular, I wish to understand invariant random subgroups in automorphisms groups of trees and of products of trees.

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Ph.D. advisor: Andreas Thom

Group stability in the context of group approximations

My research revolves around group stability in the context of (sofic) approximations. Intuitively speaking stability is a notion that stands in some sense orthogonal to approximability. If $\mathcal{C}$ is a class of groups equipped with bi-invariant metrics, then we call a group $\mathcal{C}$-*approximable* if it admits an injective homomorphism into a metric ultraproduct of elements of $\mathcal{C}$, and *stable* when every homomorphism into such an ultraproduct lifts to a homomorphism of a direct product. It is a notorious open question whether every group is approximable with respect to the class Sym of finite symmetric groups equipped with the normalized Hamming distance, i.e. sofic. The main motivation to investigate the notion of stability is the following observation: If one had a sofic, Sym-stable group, then that group admits an injective homomorphism into a direct product of finite groups and thus has to be residually finite. Hence if one were to find a non-residually finite, Sym-stable group, then that group could not be sofic. A similar approach was used recently by de Chiffre, Glebsky, Lubotzky & Thom [1] to give the first examples of groups that are not approximable with respect to the class of groups of unitary matrices equipped with the Frobenius norm.
Currently I'm working jointly with my supervisor on an analogue to a result by Peter Burton and Lewis Bowen [2] who showed the existence of a non-sofic group provided $PSL_d(\mathbb{Z})$ is not stable for some $d \geq 5$. We highly suspect that we can show the existence of a non-sofic group under the condition that a certain lattice in $Sp_{2n}(\mathbb{R})$ is stable in a very weak sense. I sincerely hope that a preprint will be available on arxiv until the conference begins.

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Higher Dehn functions of solvable Lie groups

I am a second year PhD student under the supervision of Enrico Leuzinger. In my PhD project, I study higher Dehn functions of solvable Lie groups. The 2-dimensional Dehn function of a simply connected Riemannian manifold is the smallest isoperimetric function of the manifold. Namely, for all closed curves up to a certain length, the Dehn function bounds the area of all minimal surfaces, bounded by the curve, from above. Therefore, if we have a closed curve, there is a filling of that curve whose area is smaller than the Dehn function, evaluated at the length of the curve. The Dehn function of a simply connected Riemannian manifold X measures the isoperimetric inequality in X. For each loop c in X, consider the area $\mathrm{vol}(c)$ of the minimal surface bounded by c. Then the Dehn function F quantifies the growth of such fillings relatively to the length of c:

$$F(t) = \sup_{\ell(c) \leq t} \mathrm{vol}(c).$$

Gromov showed [1] that the Dehn function of a Lie group grows at most exponentially. Furthermore, this upper bound is actually attained by $\mathrm{SOL}_3 = \mathbb{R}^2 \rtimes \mathbb{R}$, where $\mathbb{R}$ acts on $\mathbb{R}^2$ via $(x, y) \mapsto (\mathrm{e}^t x, \mathrm{e}^{-t} y)$. Furthermore, $\mathrm{SOL}_3 = \mathbb{R}^2 \rtimes \mathbb{R}$ actually attains this upper bound, where $\mathbb{R}$ acts on $\mathbb{R}^2$ via $(x, y) \mapsto (\mathrm{e}^t x, \mathrm{e}^{-t} y)$. Cornulier and Tessera showed [2] that an obstruction for all Lie groups for having polynomially bounded Dehn function arises from Lie groups "of SOL-type". Namely, a certain (generic) class of groups has an exponential Dehn function if it admits a homomorphism to a group of SOL-type. Furthermore, they show that if a Lie group does not have this obstruction and another, the so-called 2-homological obstruction, then it admits a polynomially bounded Dehn function. This obstruction, together with a so-called 2-homological obstruction, are the only obstructions. To be specific, if the Lie group do not have any of the two obstructions, it admits a polynomially bounded Dehn function. Generalizing the concept of Dehn functions, one might as well measure the area of higher dimensional fillings of higher dimensional spheres. This gives rise to the higher Dehn functions. In my research, I hope to find higher-dimensional analogues of the SOL-obstruction for those higher Dehn functions.

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Quasi-isometries of right angled groups

My research concerns quasi isometric features of right angled Artin and Coxeter groups. While in the Coxeter setting very little is known, in the Artin groups a series of papers by Huang ([1], [2], [3]) shows quite strict QI rigidity. This rigidity relies on rigidity of flats in CAT(0) Cube Complexes, a generalization of similar results in Euclidean buildings and Symmetric spaces. I am thus interested in all of these objects and their Tits boundaries, and their structure in CAT(0) Cube Complexes. I currently study the question of rigidity in the setting of QI embeddings between CAT(0) cube complexes, following results of Fisher-Whyte in the setting of symmetric spaces ([4]).

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Geometry and rigidity of relative automorphisms groups

I am interested in computing the outer automorphism group of automorphism groups of Coxeter groups and the outer automorphism group of automorphism groups of subgroups of $\mathrm{Aut}(\mathbb{F}_n)$, where $\mathrm{Aut}(\mathbb{F}_n)$ is the automorphism group of a free group of rank n.

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Group actions on hyperbolic spaces

Introduction

My research is focused on groups acting on hyperbolic spaces. This falls under the area of geometric group theory, where it is common to study a group via the geometric properties of a space on which the group acts. It is essential to consider specific types of group actions to obtain non-trivial results, since every group acts trivially on every space. Gromov realized that actions on hyperbolic spaces are useful to study algebraic, algorithmic and analytic properties of groups. He introduced the concept of *hyperbolic groups* and *relatively hyperbolic groups* in [6]. Hyperbolic groups are groups that admit geometric (proper and cobounded) actions on hyperbolic spaces, while relatively hyperbolic groups are groups that are hyperbolic modulo some collection of subgroups (called peripheral subgroups). The presence of negative curvature yields important results for the structure of these groups. However, there are several groups that belong to neither of these classes; yet admit natural actions on hyperbolic spaces; examples include the mapping class group acting on the curve complex and right angled Artin groups acting on the extension graph. Motivated by these examples, many other group actions on hyperbolic spaces have been studied, including the "acylindrical" actions. Informally, acylindricity can be thought of as a type of properness of the action of G on $S \times S$ minus a "thick diagonal". However, an action of a group on a bounded space (which is hyperbolic) is vacuously acylindrical. Thus, additional conditions are needed to rule out the degenerate cases; this is the notion of a *non-elementary* action. A group G is called *acylindrically hyperbolic* if it admits a non-elementary acylindrical action on a hyperbolic space. By [8, Theorem 1.1], this is equivalent to the condition that G is not virtually cyclic and acts acylindrically on a hyperbolic space with unbounded orbits. The term "acylindrically hyperbolic" was coined by Osin in [8]. Prior to this paper, many interesting results were obtained for groups that admit a non-elementary action on a hyperbolic space and satisfy conditions similar to acylindricity. Osin showed that all these classes of groups coincide with the class of acylindrically hyperbolic groups. It includes many examples, yet on the other hand, the theory of acylindrically hyperbolic groups is very rich (see [5, 7, 8 ,9] and references therein); and they share many interesting properties with hyperbolic and relatively hyperbolic groups.

1 Research

Comparing generating sets of a group.

One natural way to study a group G via an action is to turn the group G into a metric space. We consider the Cayley graph $\Gamma(G, X)$ of the group G corresponding to a generating set X and equip it with the word metric d_X; the group has a natural cobounded, isometric action on this space. It is easy to see that not all generating sets are equally good for this purpose: the most informative metric space is obtained when the generating set X is finite, while the space corresponding to $X = G$ forgets the group almost completely. In joint work with C.Abbott and D.Osin, we formalize this observation by ordering generating sets of G according to the amount of information retained by (G, d_X), which gives rise to a natural equivalence relation. The set of *hyperbolic structures* on G, denoted $\mathcal{H}(G)$, consists of the equivalence classes $[X]$ such that $\Gamma(G, X)$ is hyperbolic. The subset of *acylindrically hyperbolic structures* on G, denoted $\mathcal{AH}(G)$, consists of hyperbolic structures $[X] \in \mathcal{H}(G)$ such that the action of G on the Cayley graph $\Gamma(G, X)$ is acylindrical. Both $\mathcal{H}(G)$ and $\mathcal{AH}(G)$ are posets endowed with the above order. Following the standard terminology of group actions on hyperbolic spaces (see [6] for details), we easily obtain the following preliminary theorem.
For every group G, we have

$$\mathcal{H}(G) = \mathcal{H}_e(G) \sqcup \mathcal{H}_\ell(G) \sqcup \mathcal{H}_{qp}(G) \sqcup \mathcal{H}_{gt}(G),$$

where the sets of elliptic, lineal, quasi-parabolic, and general type hyperbolic structures on G are denoted by $\mathcal{H}_e(G)$, $\mathcal{H}_\ell(G)$, $\mathcal{H}_{qp}(G)$, and $\mathcal{H}_{gt}(G)$ respectively.
Elliptic structures are the easiest to classify: we always have $\mathcal{H}_e(G) = \{[G]\}$. While we do have a complete classification of lineal structures, the posets $\mathcal{H}_{qp}(G)$ and $\mathcal{H}_{gt}(G)$ have a much more complicated structure. Nonetheless, we have several interesting results in this direction; see [1, Theorem 2.3] for details. In contrast, the poset of acylindrically hyperbolic structures exhibits a much more rigid behavior, see [1, Theorem 2.6]. The proofs of these theorems make

use of hyperbolic structures on groups induced from those on hyperbolically embedded subgroups (see [1, Theorem 1.9]), which can be thought of as a particular case of the induced action map studied in [2]. While hyperbolicity of the induced action was proved in [2], we prove the acylindricity of the induced action in [1].

Hyperbolic and acylindrically hyperbolic accessibility.

We say that a group G is *$\mathcal{H}$-accessible* (respectively *$\mathcal{AH}$-accessible*) if $\mathcal{H}(G)$ (respectively $\mathcal{AH}(G)$) contains the largest element. Note that if the largest element exists, it is unique and necessarily loxodromically universal. It is easy to find examples of groups which are not $\mathcal{H}$-accessible, e.g., the direct product $F_2 \times F_2$; however, this group is $\mathcal{AH}$-accessible. Finding $\mathcal{AH}$-inaccessible groups, especially finitely generated or finitely presented ones, is more difficult. One can also ask if every acylindrically hyperbolic group is $\mathcal{AH}$-accessible. To this end we have definite answers.

Theorem. *There exists a finitely presented, acylindrically hyperbolic group that is neither $\mathcal{H}$-accessible nor $\mathcal{AH}$-accessible.*

On the other hand, many acylindrically hyperbolic groups studied are $\mathcal{AH}$-accessible.

Theorem. *The following groups are $\mathcal{AH}$-accessible.*

(a) Finitely generated relatively hyperbolic groups whose peripheral subgroups are not acylindrically hyperbolic.

(b) Mapping class groups of punctured closed surfaces.

(c) Right-angled Artin groups.

(d) Fundamental groups of compact orientable 3-manifolds with empty or toroidal boundary.

Hyperbolic structures on wreath products.

Our understanding of the poset of quasi-parabolic structures on groups is very less. Indeed, the following open questions were posed in [1].

Question. *Does there exist a group G such that $\mathcal{H}_{qp}(G)$ is non-empty and finite? Does there exist a group G such that $\mathcal{H}_{qp}(G)$ contains an uncountable chain?*

I obtain the answer to both these questions as consequences of a general theorem for wreath products $G \operatorname{wr} \mathbb{Z}$ in [3]. (stated below). In what follows, $\mathbb{S}_G$ denotes the poset of proper subgroups of G ordered by inclusion. For any group G, we will refer to the following poset as $\mathcal{B}(G)$: Two disjoint copies of $\mathbb{S}_G$ which are incomparable. i.e. every element in one copy of $\mathbb{S}_G$ is incomparable to every element in the other copy of $\mathbb{S}_G$. These two copies of $\mathbb{S}_G$ dominate a common element, which in turn dominates another element. We can now state the main theorem of [3].

Theorem. *Let G be a group. Then $\mathcal{B}(G) \subset \mathcal{H}(G \operatorname{wr} \mathbb{Z})$. Specifically, the two copies of $\mathbb{S}_G$ correspond to quasi-parabolic structures on $G \operatorname{wr} \mathbb{Z}$. The common element they dominate corresponds to a lineal structure on $G \operatorname{wr} \mathbb{Z}$; which in turn dominates the trivial structure. If $G = \mathbb{Z}_n$, then $\mathcal{B}(G) = \mathcal{H}(G \operatorname{wr} \mathbb{Z})$.*

The proof of the above theorem is a combination of the description of quasi-parabolic structures obtained (in a different language) in [4], along with elementary (but lengthy) arguments from commutative algebra. Also note that in general, the above equality does not hold. Indeed, the above equality does not even hold for every finite group G. The case when $G = \mathbb{Z}_2 \times \mathbb{Z}_2$ provides a counterexample, as shown in [3, Example 4.16]. The answer to the first question is obtained immediately. Specifically, the lamplighter groups all have an even number of quasi-parabolic structures. The answer to the second question is obtained by applying the first part of the above theorem to the group $\mathbb{F}_2 \operatorname{wr} \mathbb{Z}$. Indeed, it is easy to show that the poset of subsets of $\mathbb{N}$ ordered by inclusion embeds into $\mathbb{S}_{\mathbb{F}_2}$.

Future Research

1. Within the framework of [1] and [3], there are still several open questions. Some questions I would like to answer are the following.

Questions.

- *Is there a group G such that $|\mathcal{H}_{qp}(G)| = 1$?*
- *Which properties of the posets $\mathcal{H}(G)$ and $\mathcal{AH}(G)$ are preserved by quasi-isometries?*
- *In particular, are $\mathcal{H}$-accessibility and $\mathcal{AH}$-accessibility invariant under quasi-isometries?*

There is some evidence to the possibility that the answer might be affirmative in the case of $\mathcal{AH}(G)$. This question is also related to the larger open question of whether the property of being an acylindrically hyperbolic group is a quasi-isometry invariant.

2. Another problem that I am considering (joint with T. Fernos) deals with understanding acylindrical actions on products of hyperbolic spaces. Specifically, given a group G, hyperbolic spaces X_1 and X_2, and a map $G \hookrightarrow \mathrm{Isom}(X_1) \times \mathrm{Isom}(X_2)$, we would like to understand how the acylindricity of the action of G on $X_1 \times X_2$ relates to the actions of G on X_1 and X_2 and vice-versa. It is interesting to note that results in this direction would depend on the product structure imposed on $X_1 \times X_2$. Thus one could also consider the question as to which product structures on $X_1 \times X_2$ provide obstructions to acylindricity and which ones are compatible with acylindricity.

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Analytical and asymptotical aspects of finitely generated groups

My research interests lie somewhere between the worlds of functional analysis, geometric group theory and probabilities. Indeed, in functional analysis one has the infamous "spectral theorem for bounded self-adjoint linear operators", that generalizes on the one hand the decomposition of a finite-dimensional endomorphism as a sum of projections, and on the other hand Lebesgue's integration theory with what is known as "spectral measures".

For a finitely generated group G, with symmetric generating set $S = S^{-1}$, $|S| < +\infty$, one can look at its Cayley graph $\Gamma = \mathrm{Cay}(G, S)$ with vertex set $V = G$ and edge set

$$E = \{(g, gs) \mid g \in G, s \in S\}.$$

The adjacency matrix of Γ can be seen as an operator A on the Hilbert space $\ell^2(V)$. It is a linear, bounded self-adjoint operator. Thus the spectral theorem applies to A, and one can try to compute its spectral measure.

In particular, if we define for $v \in V$ $e_v \in \ell^2(V)$ by $e_v(w) = \delta_{vw}$, we can look at $\langle A^n e_0, e_0 \rangle$ where $0 \in V$ is the neutral element of G and $n \in \mathbb{N}$. The quantities $\langle A^n e_0, e_0 \rangle$ are exactly the number of paths of length n starting at 0, and ending at 0. In a probabilistic setting, after renormalization, one can interpret these quantities as the probabilities to return to the origin after n steps. One way to compute the spectral measure of the adjacency operator (or the Markov operator in you like probabilities more) is thus to count the number of cycles of length n in a graph.

The previous paragraphs show the relation between functional analysis, geometric group theory, and probabilities. The book [1] provides a nice introduction to spectral measures, [2] and [3] give a lot of background for probabilities on graphs. A very nice paper is [4]: the authors compute the spectral measure of the Lamplighter group $\mathbb{Z}_2 \wr \mathbb{Z}$ by using percolation theory, uncovering yet another link between spectral measures and probabilities.

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Non-positive curvature in groups

The study of hyperbolic groups, introduced by Gromov, has given us a satisfactory answer to the question: "What is a negatively curved group?" While there is yet no fully satisfactory universal notion of a nonpositively curved group, there exist many prominent categories of groups which possess features of nonpositive curvature: CAT(0) groups, systolic groups, Helly groups, bicombable groups, almost convex groups, etc.

I am broadly interested in nonpositive curvature conditions on groups and the properties of groups that satisfy them. In this pursuit I have written about groups acting on *quadric complexes*, which are a square analog of systolic groups and which generalize $C(4)$-$T(4)$ small cancellation groups. In order to construct contractible higher skeleta for quadric complexes, I defined a class of combinatorial cell complexes called *bisimplicial complexes*. The faces of bisimplicial complexes are combinatorial and yet they are not polyhedra. These faces have 1-skeleta that are complete bipartite graphs. In addition to proving their contractibility, I proved that the bisimplicial higher skeleta of quadric complexes satisfy a fixed point property for finite subgroups. [1,3]

I have also introduced a class of graphs called shortcut graphs. These are graphs satisfying a *shortcut property* for cycles: more precisely, long cycles cannot embed in them without metric distortion. I proved that hyperbolic groups, cocompactly cubulated groups and finitely generated Coxeter groups act on graphs satisfying a strong form of the shortcut property, while the standard Cayley graph of the Baumslag-Solitar group BS(1,2) satisfies a weak form of the shortcut property. I also proved that groups acting on strongly shortcut graphs have polynomial isoperimetric function while those acting on shortcut graphs have exponential isoperimetric function. In particular, they have solvable word problem. [2]

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L^p metrics on Teichmüller space

The Teichmüller space of a Riemann surface R parameterizes the possible complex structures of R up to homeomorphisms homotopic to the identity. One way to study this space is by studying quasi-conformal maps of $\mathbb{C}$, or those with bounded dilatation. One advantage of this viewpoint is that it allows us to realize the tangent space of a point in Teichmüller space as bounded Beltrami differentials, or $(-1, 1)$ forms, as they are the infinitesimal form of deformations of conformal structures. Holomorphic quadratic differentials, $(2, 0)$ forms, are then seen to be the cotangents, and there is a natural way to turn the space of holomorphic quadratic differentials on a surface into a function space. Once we have a function space we can endow it with the L^q metrics, and dualize to get the L^p, $\frac{1}{p} + \frac{1}{q} = 1$, metrics on the tangent space.

The Weil-Petersson metric on Teichmüller space arises in this way as the L^2 metric, and has been widely studied. In [1] Wolpert proves that the Weil-Petersson metric is not complete; I have shown that the same is true for the L^p metrics, $1 < p < \infty$. Naturally, the next question is to ask what the completions are. In [2] Masur shows that the completion of the Weil-Petersson metric is an augmented Teichmüller space, which contains Teichmüller spaces of noded surfaces, and extends the metric to this boundary. I am currently working on proving similar results for the general L^p metrics.

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Lattices in products of Lie groups and trees

Recently, Ian Leary and Ashot Minasyan have constructed the first examples of non-biautomatic, non-residually finite, CAT(0)-groups. The groups are also the first examples of irreducible lattices in the product space $\mathrm{Isom}(\mathbb{E}^2) \times \mathrm{Aut}(T)$, for T a locally finite regular tree. I have been working on generalising the construction to arbitrary non-compact symmetric spaces, and showing that in the more general setting, the groups still have some of the combinations of properties that Leary and Minasyan's examples enjoy.

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Isometric **orbit equivalence**

Given a free probability measure preserving (*p.m.p.*) action of $\mathbb{Z}$ on a probability space (X, μ), one can draw the graph of the dynamic: two points x and y on X are linked by an edge if and only if one can reach y by the image of x under the generator 1 or -1 of $\mathbb{Z}$. One gets a (very huge) graph on (X, μ) whose connected components are (almost surely) isomophic to a bi-infinite line. This graph is an example of huge graphs defined on probability spaces that somehow "preserve the measure μ", and that are called *graphing.* One can get the same kind of graphs from free p.m.p. actions of the group $\langle a, b \mid a^2 = b^2 = 1\rangle$, also known as the infinite dihedral group D_∞, where two points of the space D_∞ acts on are linked by an edge if and only if one is the image of the other under a or b. The graphs obtained this way are again (very huge) graphs whose connected components are (almost surely) isomorphic to a bi-infinite line. Assume that a graph(ing) G is drawn on a probability space (X, μ), and that each connected components are isomorphic to a bi-infinite line. When does the graph G comes from a free action of $\mathbb{Z}$ or D_∞ as above? A complete answer to this question is known, but the same question can be asked for other groups. For instance, when does a graphing whose connected components are (almost surely) isomorphic to a 4-regular tree comes from a free action of the free group $\mathbb{F}_2$? from a free action of the group $(\mathbb{Z}/2\mathbb{Z})^{*4} = \langle a, b, c, d \mid a^2 = b^2 = c^2 = d^2 = 1\rangle$?

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Decision problems in finitely presented groups

My research lies at the intersection of geometric and computational group theory. The main focus of my work has two directions. In the first I focus on extending the already developed algorithm (by Holt et al. in [1]), called **IsHyperbolic**, that attempts to show that a given finitely presented group is hyperbolic. Secondly, for any group presentation on which **IsHyperbolic** succeeds I am developing and proving correctness of an efficient conjugacy problem solver. The notion of a hyperbolic group was first introduced by Gromov in [5]. Suppose that G is a finitely generated group with the Cayley graph Γ endowed with its graph metric. Then G is said to be hyperbolic if there exists a $\delta > 0$ such that any triangle T in X is δ-slim: for any point p on one of the sides of T there exists a point q in the union of the other two sides with $d(p, q) < \delta$. Hyperbolic groups can be characterized in several different ways: for example, these are the groups that act on hyperbolic metric spaces. Our interests, however, focus more on their algorithmic properties. A compelling feature of hyperbolic groups is solubility of the word problem. Given a finitely presented group $G = \langle X \mid R \rangle$, we say that the word problem is soluble for G if there exists a terminating algorithm that can decide whether a given word w over $S := X \cup X^{-1}$ is equal to 1 in G. One of the ways of solving the word problem is through analysis of geometric properties of Van Kampen diagrams. It is shown in [6] that a word w over S is equal to 1 in G if and only if w is a boundary word of a Van Kampen diagram Γ, which is a connected simply-connected planar graph consisted of vertices, edges and faces, where each edge is labelled by an element of the free group F on X. Furthermore, each face of Γ has a label a product that is freely reduced without cancellation and internal faces have cyclically reduced labels equal to cyclic permutations of elements $r \in (R \cup R^{-1})$. (R^{-1} here denotes the inverse set of relators of G). Unfortunately, it is undecidable in general whether a given finitely presented group G is hyperbolic. Each method that tries to solve this decision problem (under certain restrictions) uses the following crucial fact about hyperbolic groups. A finitely generated group G is hyperbolic if and only if G admits a finite group presentation satisfying a linear *isoperimetric inequality* ([5]): for every word w equal to 1 in G there is a Van Kampen diagram with boundary label w with at most $f(|w|)$ faces, where f is a linear recursive function, called the *Dehn function*. The first method that tries to show that G satisfies a linear isoperimetric inequality uses the fact that hyperbolic groups are automatic [2]. The currently best known algorithm for proving this is the Knuth-Bendix string rewriting algorithm of [2]. The intricacy is that such automatic structures are very difficult to find. In fact, it is not known whether all hyperbolic groups have generating sets on which the algorithm will terminate. An alternative approach that can be used to prove that G has a linear Dehn function is verifying that the presentation for G satisfies the *small cancellation condition.* This is derived from the theory of [6]. These conditions state that all faces of any nontrivial van Kampen diagram Γ have a boundary with at least n edges, which is the condition $C'(1/(n-1))$ (metric) or $C(n)$ (non-metric), and such that all interior vertices of Γ have degree at least some fixed k: this is the condition $T(k)$. The procedure **IsHyperbolic** is based on a generalisation of small cancellation. One of the key ideas is to use the concept of pregroups from Stallings [8]. Since the short relators cause problems for small cancellation, the authors of [1] define a new kind of presentation for a group G, called a *pregroup presentation.* It was shown by Rimlinger in [7] that a finitely generated group H is virtually free if and only if H is the universal group $U(P)$ of a finite pregroup P. Importantly, the pregroup presentations enable one to view G as a quotient of a virtually free group $U(P)$ rather than a free group, which then allows one to ignore failures of small cancellation on the defining relators of $U(P)$. Building upon this idea the authors then define *coloured Van Kampen diagrams* over the pregroup presentations, where the relators of $U(P)$ are coloured red, and the additional relators (from a set $\mathcal{R}$) are coloured green. The second central idea of [1], the curvature redistribution (denoted as **RSym**), is derived from the Euler's formula $V-E+F=2$. Each internal edge of a coloured Van Kampen diagram Γ is initially endowed with one unit of *divergence* (negative curvature), and each vertex and internal face of Γ has curvature $+1$. We then say that **RSym** succeeds on a finitely presented group G with a given pregroup presentation when the algorithm that computes **RSym** gives rules for the allocation of this divergence to the vertices and faces of Γ in such a way that all vertices and red faces receive at least one unit, while all the internal green faces far enough from the boundary receive at least $(1+\epsilon)$ of the unit for some $\epsilon > 0$. This then shows that the contribution of the interior of the diagram to the left-hand side of Euler's equation cannot be positive, which implies that all of the curvature of any van Kampen diagram must be located near the boundary. Theorem 6.11 of [1] then gives an explicit linear bound on the Dehn function of the presentation when **RSym** succeeds on all coloured van Kampen diagrams of minimal coloured area, proving that G is hyperbolic. Moreover, under some mild conditions on the set $\mathcal{R}$ of green relators one can tests whether **RSym** succeeds in a low-degree polynomial time, thus giving a practical and implementable algorithm. My prime focus in this area is to extend this and come up with new curvature distribution schemes that enable creation of polynomial time algorithms proving hyperbolicity on a larger set of finite (pre)group presentations. Furthermore, I am working on the development and implementation of an efficient conjugacy problem solver in the situation where **RSym** succeeds. Assuming that two cyclically reduced words w_1 and w_2 represent conjugate elements in G, one can find a reduced word α over the generating set of G such that the word $W_\alpha = \alpha^{-1} w_1 \alpha w_2^{-1}$ is equal to 1 in G. Hence there is a connected simply-connected coloured diagram Γ with a boundary word W_α. Now identifying the subsets of the boundary of Γ with labels α and α^{-1} gives us an annular diagram Γ_A (which is a connected planar graph with $\mathbb{R}^2 \setminus \Gamma_A$ containing exactly two components) with outer boundary labelled by w_1 and the inner boundary labelled by w_2, which is called a *conjugacy diagram* for w_1 and w_2. Conversely, it is shown in [6] that the existence of any conjugacy diagram Γ_A for w_1 and w_2 proves their conjugacy. Choosing a reduced word α that minimizes a coloured area of Γ and using the fact that **RSym** succeeds on a given (pre)group presentation for G allows one to completely characterize the geometry of Γ_A. In particular, in all of the cases there is a path in Γ_A connecting the two boundaries that is bounded in terms of the maximal length of a green relator. This gives a quadratic time algorithm (that often runs in linear time) and as far as I know gives the best implementable conjugacy problem solver for hyperbolic groups.

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Spaces of nonpositive curvature and their boundaries

In October 2016 I started my PhD. At first I studied group actions on systolic complexes. A systolic complex is a connected, simply connected and flag simplicial complex so that any cycle of length less then 6 has a diagonal. A cycle in a complex is a subcomplex isomorphic to a triangulation of a 1-sphere. There are interesting relations between such complexes and CAT(0) spaces. In particular, 2-dimensional simplicial complexes are 6-systolic if and only if they are CAT(0). We could prove that the triangle groups (2,4,5) and (2,5,5) are not systolic, i.e. that they do not act geometrically on any systolic complex [1].
Now I am studying contracting boundaries of CAT(0) spaces, which were introduced by Charney and Sultan [2]. The contracting boundary consists of equivalence classes of contracting geodesic rays, i.e. geodesic rays that behave like geodesic rays in hyperbolic spaces. If a CAT(0) space has a nonempty contracting boundary, it has a hyperbolic-like structure. This structure can be studied by examining the contracting boundary. The contracting boundary can also be defined for CAT(0) groups. A group is CAT(0) if it acts geometrically on a CAT(0) space. The contracting boundary of a CAT(0) group is the contracting boundary of a CAT(0) space it acts on. For example, every right-angled Coxeter group acts geometrically on a CAT(0) cube complex. Thus the contracting boundary of a right-angled Coxeter group corresponds to the contracting boundary of a CAT(0) cube complex. In my PhD studies I examine contracting boundaries of particular right-angled Coxeter groups. For this I investigate contracting boundaries of corresponding cube complexes.

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Product set growth

For a finite subset U of a group G, we define its product to be $U^n = \{u_1 \cdots u_n \ : \ u_1, \ldots, u_n \in U\}$. I am interested in how $|U^n|$ behaves as $n \to \infty$. This is a more general question than the growth of balls in finitely generated groups. If G is a hyperbolic group, then it was proved in [1] that there exists $\alpha > 0$ such that every finite $U \subset G$ that is not contained in a virtually cyclic subgroup satisfies

$$|U^n| \geqslant \big(\alpha|U|\big)^{\lfloor \frac{n+1}{2} \rfloor}$$

for every $n \in \mathbb{N}$. It was also shown in [1] that if G is instead an acylindrically hyperbolic group, then there exists $\alpha > 0$ such that for a more restricted class of $U \subset G$ we have that

$$|U^n| \geqslant \left(\frac{\alpha}{\log_2^6(2|U|)}|U|\right)^{\lfloor \frac{n+1}{2} \rfloor}$$

for every $n \in \mathbb{N}$. I have shown that when the space being acted on is a quasi-tree then this logarithm term disappears, which by a result in [2] means that the logarithm term can be removed for every acylindrically hyperbolic group. My aim is to see which other groups exhibit this type of product set growth. I also have a general interest in quasi-trees, and simplicial trees with positive Cheeger constant.

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Vanishing of ℓ^2-cohomology

The Atiyah-question asks, which numbers arise as ℓ^2-betti numbers of a given group G. There are multiple conjectures by Atiyah on the answer to this question, some of which are known to be true, some are false and others are open. Consider the lamplighter-group $L = \mathbb{Z}_2 \wr \mathbb{Z} = \mathbb{Z}_2^{\mathbb{Z}} \rtimes \mathbb{Z}$ ($\mathbb{Z}$ acting by index shift) and try to compute, which numbers are ℓ^2-betti numbers of L-CW-complexes. When looking closely, you can almost see the Turing machine in L, turning the set of configurations of a Turing machine into an L-space (sort of). Using this method, it is possible to show that the question whether $\ell^2_n(C) = 0$ is not decidable for an L^2-CW-complex C and along the way find a counterexample to the original Atiyah-conjecture, that all ℓ^2-betti numbers are rational. Thus one finds a nice and unexpected connection between computational theory and algebraic topology and a hint at the scope of the general Atiyah-question. This is what I worked on before I recently began my PhD studies. Since then I have been studying ℓ^2-betti-numbers in different contexts, taking e.g. [2] as a starting point.

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Free groups and core graphs

Stallings introduced methods to study the subgroup structure of free groups using graphs. In particular, he initiated the study of core graphs and the use of a mechanism called folding. The methods developed by Stallings, together with the theory of cover spaces, give a correspondence between the category of labeled core graphs and subgroups of a free group. The first objective of my research is to extract algebraic information on the subgroup structure of free groups by studying characteristics of morphisms of the corresponding core graphs. One such attempt is a question posed by Miasnikov, Ventura and Weil. They present the idea of algebraic extensions in free groups. They conjectured that one can learn whether an extension of a subgroup is algebraic by the characteristics of the morphisms of the corresponding core graphs. Puder and Parzanchevski have found a counter-example and revised the conjecture in two ways. I have found a counter-example to both of the revised conjectures. While working on the counter example I developed method of splitting the set of homomorphisms between free groups. The second objective of my research finding more applications for these methods and developing them further.

Georgios Kontogeorgiou
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Cayley complex embeddings

Let Γ be a group, G a Cayley graph of Γ (or the Freudenthal compactification of one, if Γ is infinite) and $\sigma : G \to S^2$ an embedding of (the associated 1-complex of) G in the sphere. Then σ is said to be ($\Gamma-$)**covariant** if the canonical action of Γ on $\sigma(G)$ maps facial paths (paths that are contained in the border of a single face of $\sigma(G)$) to facial paths. More interestingly, if Γ is finitely generated, covariance can be equivalently defined as the property that the canonical action of Γ on $\sigma(G)$ can be extended to an action of Γ on S^2 by homeomorphisms ([1], Corollary 5.5).

Covariance is an important notion. In [1], it is proved that the finitely generated groups that admit a faithful, properly discontinuous action by homeomorphisms on the plane are exactly those that have a Cayley graph which admits a covariant embedding. Quite notably, covariance can be extended to a 3-dimensional setting. Suppose Γ is a finitely generated group and C is a Cayley complex of Γ. We will call an embedding $\sigma : C \to S^3$ of the (associated 2-complex of) C in the 3-sphere ($\Gamma-$)*covariant* if the canonical action of Γ on $\sigma(C)$ can be extended to an action of Γ on S^3 by homeomorphisms. My current research examines covariant embeddings of Cayley complexes. In particular, I am working to establish a 3-dimensional analogue of the above-mentioned result of [1], namely that the finitely generated groups that admit a faithful and properly discontinuous action by homeomorphisms on R^3 are exactly those that have a Cayley complex which admits a covariant embedding in S^3.

For now, I am trying to show that the former implies the latter. To do this, we begin by considering a subset K of S^3 homeomorphic to R^3 and taking Γ to act by homeomorphisms on S^3 such that the restriction of its action to K is faithful and properly discontinuous. The quotient of K induced by this action is an orbifold and its singular locus has an empty interior, which means that there is an element of it whose preimages under the natural map have trivial stabilizers (that is, there is a regular orbit). We then choose an element v of this orbit and construct non-intersecting arcs from it to all the elements of Sv (here S denotes a set of generators of Γ, excluding inverses), then take the images of these arcs under the action of Γ to construct the 1-skeleton of our Cayley complex. Similarly, we take the images of a non-intersecting fundamental domain of the 2-cells to finish the Cayley complex. Now, we have a Cayley complex mapped to S^3 in such a way that the action of Γ on its image is a restriction of the action of Γ on S^3. This map would indeed be a covariant embedding, if it was an embedding to begin with. However, there might be several places at which cells intersect at an interior point.

We can try to fix this by viewing these intersections as added edges and vertices. Now, we do have a complex embedded in S^3. Also, the action of Γ on it remains a restriction of the action of Γ on S^3. However, in the process of fixing one thing, we broke another: our altered complex might very well not be a Cayley complex of Γ. Conveniently, there is the following interesting lemma of Babai:

Lemma ([Lemma 3.6, 2]). *Let Γ be a group acting freely on a connected graph G. Then there is a connected subgraph D of G meeting each Γ-orbit at exactly one vertex, such that the contraction G/D is a Cayley graph of Γ.*

Using Smith theory, we can deduce that the points stabilized by an element of our group always form an S^0, S^1 or S^2. Keeping this in mind, we can enact subtle local alterations to transform our embedded complex to one that is freely acted upon by Γ. So, the next step is to generalize Babai's Contraction Lemma to a 3-dimensional context. Once this is done, through some elementary topological manipulations, very similar to those done in [1] for the 2-dimensional case, one could construct a covariant embedding of a Cayley complex of Γ in S^3, which would conclude the proof.

Past Research

A graph parameter p is called **self-dual** in a surface S if there is a linear function $f_S(x) = a_S x + b_S$ such that for every graph G embedded in S, $p(G^*) \leq f_S(p(G))$, where G^* denotes the dual graph of G. Moreover, if $a_S = 1$, the parameter is called **additively self-dual**. Several width parameters, like treewidth and pathwidth, have been shown to be self-dual in all 2-manifolds.

I have a deep interest in topological graph theory. In [3], Sau and Thilikos conjecture that branchwidth is an additively self-dual width parameter in all 2-manifolds, and indeed that $bw(G^*) \leq bw(G)+eg$, where eg denotes the Euler genus of the manifold in which G is embedded. For my Bachelor thesis, I proved that this conjecture holds in the case of **oriented** 2-manifolds, that is, $bw(G^*) \leq bw(G) + 2g$, g being the genus of the manifold in which G is embedded. I also generalized this result to hypergraphs embedded in oriented manifolds.

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IET, subgroups and generalizations

I am interested in topological full groups of étale groupoid acting on a Stone space. Topological full groups arise from topological dynamical systems and have been considerably studied with both a C^* algebra and group theory point of view. A lot of question about minimal Cantor system has been treated. In particular Kate Juschenko and Nicolas Monod exhibit an uncountably infinite number of non-isomorphic examples of infinite, finitely generated, simple and amenable groups. This groups are subgroups of IET of the form $\mathrm{IET}(\Lambda)$ in [1]. The group of interval exchange transformations (IET) is the group of permutation of $[0,1[$ continuous outside a finite set and right continuous which are locally a translation. I am working on abelianization of this group, some of its subgroups and its generalizations. For every Λ subgroup of $\mathbb{R}$ one can define $\mathrm{IET}(\Lambda)$ as the subgroup of IET where we impose discontinuities and translations to live in Λ. In 1981 Sah-Arnoux-Fathi exhibit an isomorphism between $\mathrm{IET}_{\mathrm{ab}}$ and $\mathbb{R} \wedge \mathbb{R}$. I generalize this result by exhibiting an isomorphism between $\mathrm{IET}(\Lambda)$ and $\Lambda \wedge_{\mathbb{Z}} \Lambda$. A way to generalize IET is to allow flips. This new group (denoted $\mathrm{IET}^{\bowtie}$) need some precautions to be define. It appears that $\mathrm{IET}^{\bowtie}$ is a perfect group. This time again I looked for subgroups of the form $\mathrm{IET}^{\bowtie}(\Lambda)$ and manage to identify its abelianization. An other generalization of IET is to remove the condition to be "locally a translation". This group PC (or $\mathrm{PC}^{\bowtie}$ when we allows flips) is a perfect group. I am currently replacing "locally a translation" by "locally an affine function" which gives a generalizations of Thompson's groups.

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Generalisations of CAT(0) cube complexes

I am a first year PhD student whose research interests lie in geometric group theory and non-commutative geometry. My Master's thesis focused on formulating the definition of a CAT(κ) space (classes of metric spaces bounded above by curvature κ) in several equivalent ways, and ultimately proving that the Gromov-Hausdorff limit of a sequence of these spaces is also CAT(κ). This CAT(κ) condition being preserved in the Gromov-Hausdorff metric is an intriguing property of these spaces, as Gromov-Hausdorff convergence is a weak type of convergence in which not many properties are conserved but this one interestingly is.

Since starting my PhD, I have been taking courses in functional analysis and topological K-theory, and have also been reading about hyperbolic and automatic groups. My PhD project is not certain at the moment, but will most likely focus on studying generalisations of CAT(0) cube complexes using tools from both geometry and analysis.

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Growth and its connections

I am interested in studying generalisations of hyperbolic groups, problems related to growth and its connections with low-dimensional topology or dynamics. The *standard growth function* $\nu_{(\Gamma,S)}(n)$ of a finitely generated group $\Gamma = \langle S \rangle$ counts the number of group elements that can be spelled by n generators. The notion of growth of a finitely generated group was introduced by A.S. Švarc (1955) and independently by J. Milnor (1968). Its motivations were mostly geometrical: for instance Švarc observed that the volume growth rate of the universal cover $\tilde{M}$ of a compact Riemannian manifold M coincides with the rate of growth of the fundamental group $\pi_1(M)$; and Milnor demonstrated that growth type of the fundamental group gives some important information about the curvature of the manifold. This was before the concept of quasi-isometry was introduced. However, an important fact of the standard growth rate is that it is a quasi-isometry invariant of groups. Now, if one defines the quantity

$$\omega(\Gamma, S) := \limsup_{n\to\infty} \frac{1}{n} \log(\nu_{(\Gamma,S)}(n))$$

it turns out that $\omega(\Gamma, S)$ is not a quasi-isometric invariant; only the fact that $\omega(\Gamma, S)$ is positive or not is.

Definition. 1. *If $\omega(\Gamma, S) > 0$ we say Γ has exponential growth, otherwise if $\omega(\Gamma, S) = 0$ we say Γ has subexponential growth.*

2. *If*

$$\omega(\Gamma) := \inf_S \omega(\Gamma, S) > 1$$

where the infimum is taken over all finite systems of generators, we say Γ has uniform exponential growth.

It is known that there are groups that have exponential growth but not uniform exponential growth. However, all of the examples given so far are not finitely presented. So, for instance, a conjecture in the field is the following one:

Conjecture. *Finitely presented groups of exponential growth have uniform exponential growth.*

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Topological properties of algebraic groups

I research various topological structures on groups, I am particularly interested in algebraic groups over C_p - the completion of the algebraically closed field $\overline{\mathbb{Q}_p}$ which is a complete but not a local field. In the case of algebraic groups over local fields it was shown by H.Omori [1] for $\mathbb{R}$ and later by Bader-Gelander [2] for any local field that semi simple groups have the property that any continuous homomorphism from the group into any other topological group has a closed image, I am working on extending this result for semi simple algebraic groups over C_p. I do this by looking at topological groups in terms of there uniform structure rather then their topological structure.
Also I am interested in applications of ergodic theory in algebraic groups. I am currently studying this subject and I find it fascinating.

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Group equations

I'm working under the supervision of Laura Ciobanu on understanding the solution sets to equations in groups. Let G be a group. An *equation* in G is a word ω over $G \cup \{X_1,\ X_1^{-1} \ldots,\ X_n,\ X_n^{-1}\}$, where $X_1,\ X_1^{-1},\ \ldots,\ X_n,\ X_n^{-1}$ are symbols not used in G, and is usually denoted $\omega = 1_G$. A *solution* to such an equation is a tuple $(g_1,\ \ldots,\ g_n)$ of elements of G, such that replacing each X_i with g_i and each X_i^{-1} with g_i^{-1} results in a word that is equal in G to 1_G. The *solution set* to an equation is the set of all solutions. For example, equations in $\mathbb{Z}$ correspond to linear equations in integers, where we only consider integer solutions to be valid. If G is a group, generated by a finite set Σ, with a normal form $\eta\colon G \to (\Sigma \cup \Sigma^{-1})^*$, we can think of the solution set to an equation ω as a language over $\Sigma \cup \Sigma^{-1} \cup \{\#\}$, where # is any symbol not used, by defining

$$L_\omega = \{(g_1\eta)\# \cdot \#(g_n\eta) \mid (g_1,\ \ldots,\ g_n) \text{ is a solution to } \omega\}.$$

By showing that L_ω is a "nice" type of language, certain algorithmic properties can be understood about solving equations. For example, in [1], Ciobanu, Diekert and Elder proved that L_ω is EDT0L in any finitely generated free group, with the reduced word normal form. This gives a bound on the space complexity of computing the solutions. Currently, I am interested in understanding the languages L_ω, where ω is an equation in a nilpotent group of class 2, or a Baumslag-Solitar group BS$(1,\ n)$.

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Bredon cohomology and classifying spaces

I am a first-year PhD student in Southampton working with Nansen Petrosyan. I study Bredon cohomology of groups relative to families of subgroups. This is a generalization of ordinary group cohomology. There is a corresponding notion of Bredon cohomology for G-spaces. Then the Bredon cohomology of a group G relative to a family $\mathcal{F}$ is isomorphic to the Bredon cohomology of the classifying space $E_{\mathcal{F}}(G)$, that is a terminal object in the G-homotopy category of G-CW-complex with isotropy groups in $\mathcal{F}$. Note that for the trivial family $\mathcal{TR}$ consisting only of the trivial subgroup, a model for $E_{\mathcal{TR}}(G)$ is given by EG. Families of particular interest are the families $\mathcal{FIN}$ of all finite subgroups and $\mathcal{VCY}$ of all virtually cyclic subgroups. The associated classifying spaces $E_{\mathcal{FIN}}(G)$ and $E_{\mathcal{VCY}}(G)$ appear in the Baum–Connes conjecture for topological K-theory of group C^*-algebras and the Farrell–Jones conjecture for algebraic K- and L-theory of group rings, respectively.

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The combinatorics of hyperbolic Coxeter groups

The subject of my current research are the combinatorics of hyperbolic coxeter groups within the context of my PhD studies since last November. More precisely, I focus on reflection length in non-affine infinite Coxeter groups.
For a Coxeter group W with generating set S the conjugates of the generators are called *reflections*. For an element $w \in W$ the minimal number $l_R(w)$ of reflections r_i that are needed to express w (e.g. $w = r_1 \cdots r_k$) is called *reflection length*. It is known that reflection length as a function $l_R : W \to \mathbb{N}$ is bounded on affine and unbounded on hyperbolic Coxeter groups (see [1]). Together with others one of my supervisors Petra Schwer established a formular to compute reflection length in the affine case (see [2]). The objective of my current research is to study the asymptotic behaviour of l_R and find repetitive patterns and prove structural results about the reflection length function in the hyperbolic setting: Even though the reflection length is unbounded, it already seems difficult not only to cumpute it but also to find elements with great reflection length in hyperbolic Coxeter groups.

Previously, I wrote my Bachelor and Master thesis on the local structure of locally compact groups and on a geometric proof of the strong Hanna Neumann conjecture by Igor Mineyev. The former deals mainly with local splitting properties of these groups and is strongly related to a popular paper by Iwasawa (see [3]).

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Algorithmic problems for $Out(F_n)$

I am interested in algorithmic problems such as the word and conjugacy problem for $Out(F_n)$. I study certain dynamics of the $Out(F_n)$ action on outer space and corresponding bordifications. Specifically, given two points in the free splitting complex or in the spine of outer space, I am interested in understanding what makes some connecting paths more efficient than others.

Other research interests of mine include interval exchange maps on foliated compact surfaces with boundary. In particular, I am interested in what information can be extracted from iteratively folding indivisible Nielsen paths corresponding to stabilized train track maps.

Maria Magnani
Heildelberg
Ph.D. advisor: Beatrice Pozzetti

Character variety and Teichmüller space

I just started a PhD in Heidelberg in the research group of Anna Wienhard and Maria Beatrice Pozzetti. In my Master thesis I studied a paper of Ballas, Danciger and Lee on convex projective geometry. In particular the main question was the following: if a closed three-manifold which has a geometric decomposition (in the sense of the Thurston decomposition) containing only hyperbolic pieces, must the manifold admit a properly convex projective structure? Following the work of Ballas, Danciger and Lee I constructed a concrete example where the question has positive answer, and this was done by gluing convex projective three manifolds around hyperbolic structures. In my PhD my topics of interest are character variety and Teichmüller space.

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Characterizations of stable subgroups

Suppose G is a finitely generated group. A subgroup $H \leq G$ is said to be *stable* if H is undistorted in G, and for all $K \geq 1$ and $C \geq 0$, there exists $R = R(K, C) \geq 0$ satisfying the following: for any pair of (K, C)-quasigeodesics of G that share common endpoints in H, each is contained in the R-neighborhood of the other. In the case that $G = \mathrm{Mod}(S)$, where S is a connected, orientable surface, we have two equivalent characterizations of stable subgroups. Durham and Taylor [1] showed that a subgroup $H \leq \mathrm{Mod}(S)$ is stable if and only if it is convex cocompact. Independently, Hamenstädt [2] and Kent-Leininger [3] showed that a finitely generated subgroup $H \leq \mathrm{Mod}(S)$ is convex cocompact if and only if some (any) orbit map of H to the curve graph is a quasi-isometric embedding. I am currently exploring other situations in which a finitely generated group acts on a hyperbolic space, and trying to determine whether stability in these cases is equivalent to some orbit map being quasi-isometrically embedded in the hyperbolic space.

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Semisimple locally compact groups and duality

In [1] and [2] Caprace, Reid and Willis studied totally disconnected locally compact groups by constructing Boolean lattices inside the group. This was combined with Stone's Duality theorem and the natural action of the group on these lattices via conjugation to obtain some deep results about the topology of the group. For a topological group these lattices are not always boolean, but some still form semilattices. This can be used to construct a functor from the category of topological groups to the category of semilattices. I want to study this functor as an analog of the Lie functor for Lie-groups, which is especially powerful for semisimple Lie groups. Therefore I want to generalize the definition of a semisimple Lie group to a locally compact topological group and hope to use this functor (and its "cousin") to obtain similar theorems. This could be a way to merge the theories revolving around connected Lie groups and totally disconnected groups adding further information to the structure of (semisimple) locally compact groups.

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Weak Modularity

Coxeter groups with large enough torsion coefficients are known to satisfy non-positive curvature properties such as being cubulated and systolic. However, there are known examples of Coxeter groups which are not cubulable or systolic. In particular, the 1-skeleta of the associated Coxeter complexes are not median or systolic. For such a class of Coxeter complexes, I have proven weak modularity, a non-positive curvature condition generalizing median and systolic graphs.

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Uniform exponential growth in groups with non-positive curvature

A group is said to satisfy the strong Tits alternative if any finitely generated subgroup is either virtually abelian or contains a free subgroup. Many natural classes of non-positively curved groups have been showed to satisfy this alternative including: groups acting properly on CAT(0) cube complexes, hierarchically hyperbolic groups, and free-by-cyclic groups. Given a particular generating set, if a free subgroup exists we want to understand explicit generators. Producing such elements with uniformly bounded word length implies gives uniform exponential growth of the group. To show uniform exponential growth, however, it suffices to produce a basis for a free semigroup. My research attempts to quantify the Tits alternative for groups with various notions of nonpositive curvature, including cubical groups in joint work with Gupta and Jankiewicz, hierarchically hyperbolic groups in joint work with Abbott and Spriano, and free-by-cyclic groups in joint work with Kropholler and Lyman.

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Finitely generated normal subgroups of Kähler groups

A complex manifold is a differentiable manifold endowed with an atlas such that the transition functions are holomorphic. A Kähler manifold is a complex manifold endowed with a hermitian metric such that its imaginary part is a non-degenerate closed 2-form. A group is called a Kähler group if it can be realized as the fundamental group of a compact Kähler manifold, a classical reference on this subject is [1]. Similarly, a group is called a surface group if it can be realized as the fundamental group of a closed oriented surface of genus greater than one. Some examples of Kähler groups are finite groups, abelian groups of even rank and surface groups. My general interests are connections between Kähler groups, geometric group theory and complex differential geometry. During my PhD I have worked with particular situations mixing Kähler groups and the Bass-Serre theory. My research project is currently focused on the study of Kähler groups admitting a finitely generated group as a normal subgroup. This situation can be expressed by a short exact sequence

$$1 \to N \to G \to Q \to 1,$$

where G is a Kähler group and N is a finitely generated group. A particular situation I am studying is the case when N is a surface group. For my forthcoming work, I am interested in (a mixture of) the following situations:

1. Study the short exact sequence

   $$1 \to N \to \pi_1(X) \to \pi_1(Y) \to 1$$

   induced by a surjective holomorphic map with connected fibers $f : X \to Y$, where X is a compact Kähler manifold and Y is a compact complex manifold.

2. Kähler groups having exoctic finiteness properties. Recall that a group G has finiteness type $\mathcal{F}_r$ if there exists a CW complex $K(G, 1)$ with finite r-skeleton.

3. Kähler groups acting on hyperbolic spaces.

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QI rigidity and relative ends

In [1], Schwartz proves his classical rigidity result for rank one lattices. Then Frigerio, Lafont, and Sisto prove in [2] a more general rigidity result for high-dimensional graph manifold pieces. Namely, they demonstrate the QI rigidity of the class of groups of the form $\pi_1(M) \times \mathbb{Z}^d$, where M is any complete finite-volume hyperbolic n-manifold, $n \geq 3$. I am interested in whether QI rigidity holds for the class of groups of the form $\pi_1(M) \times N$, where M is as before and N is any simply connected nilpotent Lie group. I am also interested in the theory of relative ends, which is largely driven by Kropholler's conjecture about group splittings. In [3], Kropholler and Roller introduce an algebraic end invariant $\tilde{e}(G, S)$ for pairs of groups $S \leq G$. While the number of ends of a finitely generated group is either 0, 1, 2, or infinity, $\tilde{e}(G, S)$ can take on any positive integer value. However, Kropholler and Roller prove that if S has infinite index in $\mathrm{Comm}_S(G)$, the commensurator of S in G, then in fact $\tilde{e}(G, S)$ is either 0, 1, 2, or infinity. The definition of the algebraic end invariant $\tilde{e}(G, S)$ relies only on the natural action of G on itself, and so the definition may be extended for (sufficiently nice) actions of G on a general space. I am investigating the extent to which the results of Kropholler and Roller continue to hold for this more general end invariant.

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Isometric actions on metric spaces with non-positive curvature

I am interested in understanding isometric actions on metric spaces with some kind of non-positive curvature. I could say there are two main branches in my research.

1 Dynamical invariants for NPC spaces

Joint stable length and the Berger-Wang property

If (X, d) is a (Gromov) hyperbolic metric space and $f \in \mathrm{Iso}(X)$ is an isometry, then it is well known that its *stable length* $d^\infty(f) = \inf_{n\geq 1} d(f^n x, x)/n$ is independent of $x \in X$ and determines whether f is loxodromic. In [3], and motivated by analogous notions for sets of matrices, I generalized this invariant and defined the *joint stable length* (j.s.l.) of a finite subset $S \subset \mathrm{Iso}(X)$ by

$$\mathfrak{D}(S) := \inf_{n\geq 1} \max_{g\in S^n} \frac{d(gx, x)}{n},$$

where S^n denotes the set of products of n elements in S. While it is an exercise to see that the inequality $\mathfrak{D}(S) \geq \sup_{g\in S^n} d^\infty(g)/n$ holds for any n and for any (non necessarily hyperbolic) metric space (X, d), the converse inequality depends non trivially on the geometry of X.
We say a metric space (X, d) has the *Berger-Wang* property if for any finite subset $S \subset \mathrm{Iso}(X)$,

$$\mathfrak{D}(S) = \sup_{n\geq 1} \sup_{g\in S^n} \frac{d^\infty(g)}{n}. \tag{1}$$

Again, this definition is motivated by its similarity to the Berger-Wang identity, a property about the growth rate of matrix products. In simple terms, in spaces satisfying the Berger-Wang property, the maximal linear growth rate of any subset S of isometries (l.h.s. of (1)) can be recovered with arbitrary accuracy by looking at finitely many compositions of isometries in S (r.h.s.). One of the main results in [3] is the following

Gromov hyperbolic spaces have the Berger-Wang property.

Moreover, if (X, d) is δ-hyperbolic, $x \in X$ and $S \subset \mathrm{Iso}(X)$ is finite, then

$$d(S^2, x) \leq d(S, x) + \frac{\sup_{f \in S^2} d^\infty(f)}{2} + 6\delta. \tag{2}$$

This result was later re-obtained by Breuillard-Fujiwara [1], who also proved that symmetric spaces of non-compact type have the Berger-Wang property, and that Euclidean spaces $\mathbb{R}^d$ don't have it if $d \geq 4$. This is interesting since it reflects that the j.s.l. is able to distinguish between flat and non-trivial non-positive curvature, at least in higher dimensions.

I'd like to find more families of metric spaces satisfying the Berger-Wang property (maybe mapping class groups, or cocompact CAT(0) cube complexes), specifically those satisfying an inequality similar to (2), since the methods from Breuillad-Fujiwara are unable to deduce it. In a forthcoming paper, and refining the techniques from [2], I prove an analogue of (2) for symmetric spaces of non-compact type.

An ergodic approach: the Morris property

To compute the j.s.l. of $S \subset \mathrm{Iso}(X)$ we must consider the maximum linear growth rate of sequences of products over *all* possible sequences of isometries in S. *But what can we say by considering only a subset of such sequences?*

We codify this situation in a probability space $(\Omega, \mathcal{F}, \mu)$ with a measure-preserving map $T : \Omega \to \Omega$, and a measurable map $A : \Omega \to \mathrm{Iso}(X)$ inducing the maps $A^n(\omega) = A(\omega)A(T\omega)\cdots A(T^{n-1}\omega)$ for $n \geq 1$ and $\omega \in \Omega$ (we endow $\mathrm{Iso}(X)$ with the compact-open topology). We call A a *(metric) cocycle*, and say A is integrable if $\omega \mapsto d(A(\omega)x, x)$ is integrable for some $x \in X$.

Metric cocycles arise naturally in a variety of contexts regarding random processes, and resembles some of the properties of the extensively studied linear cocycles. In particular, Kingman's theorem implies that when μ is ergodic, the *drift of* A

$$\mathrm{dr}_\mu(A) := \lim_{n\to\infty} \frac{d(A^n(\omega)x, x)}{n}$$

is finite and constant μ-a.e. (if you are familiar with non-commutative ergodic theory, this should remind you the *upper Lyapunov exponent* of a linear cocycle). As a motivating example, for a finite set $S \subset \mathrm{Iso}(X)$ consider the compact space $\Omega := S^{\mathbb{N}}$ and the shift map $T : \Omega \to \Omega$ given by $T(g_0, g_1, g_2, \dots) = (g_1, g_2, g_3, \dots)$. If we define the cocycle $A : \Omega \to \mathrm{Iso}(X)$ as $A((g_0, g_1, \dots)) = g_0$, by standard ergodic theory we can recover the j.s.l. of S according to

$$\mathfrak{D}(S) = \sup_{\mu \in \mathcal{E}_T} \mathrm{dr}_\mu(A),$$

where $\mathcal{E}_T$ denotes the set of T-invariant ergodic Borel probability measures on Ω. We can also check that if $g = g_0 g_1 \cdots g_{n-1}$ with $g_i \in S$ then

$$d^\infty(g)/n = \mathrm{dr}_{\mu_p}(A),$$

with μ_p being the probability measure supported on the T-orbit of $p = (g_0, g_1, \dots, g_{n-1}, g_0, g_1, \dots)$. This implies that the Berger-Wang identity (1) follows from the approximation of $\mathfrak{D}(S)$ by drifts of ergodic measures supported on periodic orbits. For a general measurable dynamical system there may not exist periodic orbits, so instead of approximating the drift of a (non-necessarily maximizing) measure by measures of finite support, we would like approximate by stable lengths. We say that a metric space (X, d) has the *Morris property* if for any ergodic dynamical system $(\Omega, \mathcal{F}, \mu, T)$, and for any integrable cocycle $A : \Omega \to \mathrm{Iso}(X)$ the identity

$$\mathrm{dr}_\mu(A) = \limsup_{n \to \infty} \frac{d^\infty(A^n(\omega))}{n} \tag{3}$$

holds for μ-a.e. $\omega \in \Omega$. As before, this definition is inspired by the work of Ian Morris on generalized versions of the Berger-Wang identity for linear cocycles, and in fact it is not difficult to prove that the Morris property implies the Berger-Wang property. By a refined version of (2), in [2] I proved the following:

Gromov hyperbolic spaces have the Morris property.

I also showed that the Morris property fails in $\mathbb{R}^d$ for any $d \geq 2$, and hence the Morris property is *strictly* more sensitive to non-positive curvature than the Berger-Wang property. In a forthcoming paper, and by extending the results of [2] I also prove:

Symmetric spaces of non-compact type have the Morris property.

2 Relatively Hyperbolic Groups and special cube complexes

CAT(0) cube complexes have played a prominent role in the last decade, by their connection with geometric topology and group theory. In particular, the class of special cube complexes introduced by Wise and his collaborators was a key point in Agol's proof of the virtually Haken conjecture, who proved that compact NPC cube complexes with hyperbolic fundamental group have special finite-sheeted covers. Recently, I've been thinking about compact NPC cube complexes with relatively hyperbolic fundamental groups. For the moment, I've been able to prove the following:

If X is a compact NPC cube complex with hyperbolic fundamental group relative to virtually abelian groups, then X is virtually special.

This result already encompasses several interesting examples, such as limit groups and fundamental groups of finite-volume hyperbolic 3-manifolds. In the future I expect the same conclusion to be true in the case X is sparse, or in the compact case but only assuming $\pi_1(X)$ is hyperbolic relative to virtually compact special groups. The compact case with virtually compact special peripherals would follow from the following conjecture

If X and Y are homotopy equivalent compact NPC cube complexes and X is virtually special, then Y is also virtually special.

The strong separability properties of polycyclic groups imply the conjecture when $\pi_1(X)$ is virtually abelian, and by results of Haglund-Wise it is also true when $\pi_1(X)$ is hyperbolic. Right-angled Artin groups would be the first testing-cases to check.

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Large-scale geometry of Lie groups

One vast current problem in the geometric theory of real connected Lie groups is their classification up to quasi-isometry (QI). Many quasi-isometric invariants for such a group G are known and studied [4]. They include the asymptotic cones $\mathrm{Cone}(G)$, that are informally "pictures of the group as seen from the infinity" (with all the information coming along, e.g. $\pi_1(\mathrm{Cone}(G))$), but also the growth and the filling invariants, among which the Dehn function δ_G, which measures the difficulty to fill loops of given length in the group in an asymptotic way. Through what they retain on the large-scale geometry of groups, those invariants are sometimes related; for instance if a Lie group G has simply connected asymptotic cones, then δ_G is bounded by a polynomial function. Further, if asymptotic cones are additionally locally compact then one can bound from above the degree of growth of δ_G [3], and even estimate exactly the growth [7], from the knowledge of a single asymptotic cone.

Quasi-isometries are not the only maps that preserve all the features of asymptotic cones, though: so do Sublinear BiLipschitz Equivalences (SBE) [1]. These equivalences occur quite naturally between pairs of non-isomorphic Lie groups provided that they have sufficiently close algebraic structure; they preserve coarse structures, though rather unusual ones [2]. The classification of Lie groups up to SBE is necessarily less fine than what we know or expect from the QI classification; nevertheless some invariants can be derived from quasi-conformal analysis (in a generalized sense) on the boundaries of Gromov-hyperbolic Lie groups [6]. With such techniques, some partial progress can be expected towards the classifications of hyperbolic Lie groups up to QI and SBE, as well as an improved understanding of the large-scale geometry of such groups.

My current research projects include the search and computation of further invariants, especially for groups of polynomial growth. In an application of this circle of ideas, following Cornulier and in joint work in progress with C. Llosa Isenrich and R. Tessera, we exhibit pairs of Lie groups that have bi-Lipschitz simply connected locally compact asymptotic cones, but different Dehn functions [5]. I also expect the SBE may play a role beyond the realm of Lie groups, as quasi-isometries do, but possibly in different situations; one of those being the first passage percolation on the Cayley graphs of finitely generated groups.

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Hyperbolic spaces for the outer automorphism group of Baumslag-Solitar groups

The automorphism group $Out(F_N)$ of a free group has been studied and already many results have been found thanks to hyperbolic spaces such as the complex of free factors. As Baumslag-Solitar groups share some properties with free groups, the aim of my research is to try to extend some of these results to automorphism groups of Baumslag-Solitar groups. A Baumslag-Solitar group is defined as

$$BS(p,q) := \langle a,t | ta^p t^{-1} = a^q \rangle$$

for some integers p, q. One can build a *deformation space*, which is an analogue of the outer space of Culler-Vogtmann. It is a powerful tool for studying the automorphism group. This can also be done with generalized Baumslag-Solitar groups, which are obtained by taking successive HNN extensions and amalgamated free products of cyclic groups above cyclic groups $\mathbb{Z}$. The existence of irreducible automorphisms and train track maps in the generalized Baumslag-Solitar group case is one of the questions I am interested in. Some irreducible automorphisms do admit a train track map. I would like to use this fact to build an action of my outer automorphism groups on a hyperbolic space.

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Minimal volume entropy

I am a second year PhD student working under the supervision of Enrico Leuzinger. My research for my PhD project is focused on volume entropy of various structures. The volume entropy of a compact Riemannian manifold is the exponential growth rate of the volume of a metric ball in its universal cover. This is an asymptotic invariant, not depending on the base point of the ball. It is of great interest due to its relation to the fundamental group and systoles of a Manifold [1]. For this purpose one is especially interested in the metric minimizing the volume entropy of a given manifold under all volume one metrics. Moreover, there is a connection with the topological entropy of the geodesic flow which is a measure of the complexity of a dynamical system. A. Manning [2] showed that the topological entropy is an upper bound for the volume entropy and these values coincide if the manifold has non-positive sectional curvature. Besides, S. Lim [3] adapted this definition to graphs and received a strict lower bound for the volume entropy of a given graph for a volume one metric. Moreover, she states a length assignment of the edges, realizing it. In my research I try to extend the definitions among other things to hypergraphs, simplicial complexes and buildings. Furthermore, I am interested in lower bounds on the volume entropy under certain restrictions.

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Embeddings in finitely presented groups

The celebrated Higman embedding theorem states that every recursively presented group is the subgroup of some finitely presented group. This theorem has been made more precise in several cases: it is known that such an embedding can preserve solvability of the word and conjugacy problems, and a theorem by Baumslag shows that any finitely generated metabelian group embeds in a finitely presented metabelian group. On the other hand, it is known that all finitely presented residually finite groups have solvable word problem, while this is not the case for recursively presented residually finite groups, thus one cannot hope for a Higman-type theorem for residually finite groups. My research focuses on finding obstructions to the existence of Higman embedding theorems for other classes of groups.

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Hierarchically hyperbolic spaces, cubulating groups and RAAGs

My research is centered on the class of hierarchically hyperbolic groups, a notion that was recently introduced by Behrstock, Hagen and Sisto in a series of papers [1] and [2]. These groups - and the related hierarchically hyperbolic spaces - are far-reaching generalisations of hyperbolic groups and hyperbolic spaces, mapping class groups, Teichmuller spaces, cubulable groups, and right-angled Artin (or Coxeter) groups. Hierarchically hyperbolic groups and spaces offer a common viewpoint and a machinery adapt to the study of these seemingly distant notions, recovering results known to hold for (some of) the mentioned families and, at the same time, extending and uniformising these results to a broader context. As the names suggest, these groups are constructed from hyperbolic pieces in a hierarchical way. This geometric nature reflects into strong algebraic and asymptotic properties: hierarchically hyperbolic groups are finitely presented, they satisfy a (at most) quadratic isoperimetric inequality, they are coarse median and have finite asymptotic dimension. The main context of my research is to better understand certain classes of hierarchically hyperbolic groups and relate them even closer to cubulable groups and right-angled Artin groups.

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Horoboundary theory for Cayley graphs

For a finitely generated group G with finite generating set S, consider the Cayley graph $C_{G,S}$ as a metric space with the graph metric d. For $f : G \to \mathbb{R}$ denote $\nabla f = \nabla_S f := \sup\{|f(gs^{-1}) - f(g)| : s \in S, g \in G\}$. Consider $L(G) = \{f : G \to \mathbb{R} : \nabla f = 1, f(1) = 0\}$ which we shall refer to as the space of 1-Lipschitz (nonconstant) functions. There is an embedding of G into $L(G)$ by letting $g \mapsto b_g$ and b_g is defined as follows:

$$b_g(h) = d(g, h) - d(g, 1)$$

If we take $L(G)$ to be a topological space with respect to pointwise convergence (which is the same as covergence on compact sets, since G is discrete), then by Arzela-Ascoli theorem it is a compact space and hence G has a compact closure in it. The boundary of G is called the *horoboundary* ∂G. Note that G acts on $L(G)$ in the following way:

$$g.f(h) = f(g^{-1}h) - f(g^{-1})$$

If $f \in L(G)$ is fixed by this action, then it is a group homomorphism from G to the additive group $(\mathbb{R}, +)$. More generally, if the orbit of f is finite then by the orbit-stabilizer theorem it is a virtual homomorphism, i.e. $f : H \to \mathbb{R}$ is a homomorphism where $H = stab(f)$, $[G : H] < \infty$. Also, ∂G is invariant under the action of G, so when G is amenable, there exists an invariant probability measure on ∂G, and if this measure happens to be atomic then any atom f has a finite orbit, and thus is a virtual homomorphism.

The horoboundary can also be studied from the perspective of geodesic rays in the Cayley graph. Given any sequence $\{g_n\}_{n=1}^{\infty} \subseteq G$ there is the corresponding sequence in $L(G)$, which has a convergent subsequence by compactness. If the limit lies in ∂G then the original sequence in G must go to infinity with respect to the graph metric. Thus we can think of paths in $C_{G,S}$ which go to infinity, as accumulating at points in the horoboundary. Of particular interest are geodesic rays, that is, isometries of the form $\gamma : \mathbb{N} \to C_{G,S}$, so that $\|\gamma(t) - \gamma(s)\| = |t - s|$ for all t, s. It can be shown that if γ is a geodesic ray then $\{b_{\gamma(t)}\}_{t=0}^{\infty}$ converges to some $f \in \partial G$, which is called the limit point of γ. The subspace of ∂G consisting of all limit points of geodesic rays is closed and invariant. Thus, if G is amenable then there is an invariant probability measure on this compact subspace. This allows one to define a notion of "random geodesic rays", up to equivalence of rays that converge to the same boundary point. We are interested in studying the properties of these "random geodesics", such as the expected number of intersections between two independent random geodesic rays. We hope to relate these properties to the geometry of the Cayley graph and to the algebraic properties of the group itself.

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Matrix groups, Thompson groups, and 3-manifolds

I am a postdoc in the Geometry Group of Petra Schwer in Magdeburg. I enjoy investigating groups via combinatorics (e.g. presentations, algorithms) and geometry/topology (e.g. (co)homology, geometric invariants, actions on nice spaces), with recent focus on matrix groups and relatives of R. Thompson's groups $F \subset T \subset V$. In the **linear front**, an ancient problem regarding arithmetic lattices is to compute their finiteness length. This is a quasi-isometry invariant which reveals (co)homological information about the group and spaces on which it acts nicely. Major works (with landmarks by Borel–Serre, Abels, Bux, and Bux–Köhl–Witzel) have established the finiteness length of S-arithmetic subgroups of semi-simple groups and of their Borel subgroups. A natural follow-up problem is to compute the finiteness length of parabolics and proper subgroups of Borel groups. In [2,3] I gave bounds on the finiteness length of many groups lying in those classes. My current and future directions concerning arithmetic lattices and associated spaces include Reidemeister classes (j/w P. M. Lins de Araujo), filling functions, and local-to-global rigidity of Euclidean buildings (j/w P. Schwer). As for the **Thompson family**, the groups of Thompson and their relatives play a central role in geometric group theory for their peculiar properties. Main problems in the area concern the finiteness length, amenability, subgroup structure, and algorithmic questions. Kai-Uwe Bux and I proved that the braided variant of V has decidable conjugacy problem using 3-manifold-topology [1]. In the future I intend to investigate dynamics for braided V, Reidemeister classes of Monod's group (j/w A. Santos de Oliveira Tosti) and conjugacy classes and classifying spaces for irrational-slope Thompson groups (j/w B. Nucinkis). As for **3-manifolds**, I am currently looking into the algorithmic recognition of spatial graphs in the 3-sphere (j/w S. Friedl, L. Munser, J. Quintanilha).

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Superrigidity of measurable cocycle of complex hyperbolic lattices into $SU(m,n)$

In [1] Zimmer proved a super-rigidity result for lattices in connected simple Lie groups of rank at least 2. On the other hand Pozzetti proves in [2] that any maximal Zariski dense representation of a complex hyperbolic lattice into $SU(m,n)$ (with $1 < m < n$) is the restriction of a representation of the whole group $\mathrm{PU}(p,1)$.
The aim of my research is to extend the result of Pozzetti to measurable cocycles, namely measurable maps

$$\Gamma \times X \to \mathrm{SU}(m,n)$$

stisfying a cocycle condition in the sense of Eilemberg-MacLane where Γ is a lattice in $\mathrm{P\ U}(p,1)$ and X is a space equipped with a Borel measure and with a measure preserving Γ-action. Following Moraschini and Savini in [3] we introduce the Toledo invariant associated to a measurable cocycle that allows us to define maximal cocycles. Hence, under suitable hypotesis of Zarisky density of the image, we try to prove that each measurable maximal cocycle is equivalent to the cocycle induced by a representation

$$\rho : \mathrm{PU}(p,1) \to \mathrm{SU}(m,n).$$

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Subset currents on hyperbolic groups and surfaces

My area of research is geometric group theory and hyperbolic geometry. Specifically, I study the space $\mathrm{SC}(G)$ of *subset currents* on a (word-)hyperbolic group G, which was introduced by Kapovich and Nagnibeda in [1] as a generalization of the space $\mathrm{GC}(G)$ of *geodesic currents* on a hyperbolic group G. For a closed hyperbolic surface Σ, the space $\mathrm{GC}(\pi_1(\Sigma)) =: \mathrm{GC}(\Sigma)$ was introduced by Bonahon and has been used successfully in the study of the mapping class group and the Teichmüller space of Σ. The notion of measured laminations introduced by Thurston seems to be more well-known than geodesic currents. The space of measured laminations on Σ is a measure-theoretic completion of the set of weighted simple closed geodesics on Σ, whereas $\mathrm{GC}(\Sigma)$ is a measure-theoretic completion of the set of weighted closed geodesics (not necessarily simple) on Σ. One of the reasons that such completions are useful is that some of "invariants" with respect to closed geodesics, such as length and intersection number, can be extended to a continuous functional on the space of measured laminations and $\mathrm{GC}(\Sigma)$. In [1] Kapovich and Nagnibeda investigated the space $\mathrm{SC}(F)$ of subset currents on a free group F of finite rank, and proved that $\mathrm{SC}(F)$ is a measure-theoretic completion of the set of weighted conjugacy classes of finitely generated subgroups of F. Note that a (non-oriented) closed geodesic on Σ corresponds to a conjugacy class of a "cyclic" subgroup of $\pi_1(\Sigma)$. They also proved that the Euler characteristic of the conjugacy class of a finitely generated subgroup of F can be extended to a continuous functional on $\mathrm{SC}(F)$. In [2] I extended a product of two finitely generated subgroups of the free group F, which appears in the inequality of the Strengthened Hanna Neumann Conjecture, to an intersection functional on $\mathrm{SC}(F)$ and generalized the inequality. In [3] I proved the completion theorem and extended some interesting invariants to continuous functionals in the case of subset currents on a compact hyperbolic surface.

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Non-positively curved groups and manifolds

My research program started with understanding nonpositively curved manifolds and has now evolved into studying other areas like link homotopy, boundaries of groups, and arithmetic hyperbolic manifolds.

1 Non-positively curved manifolds

Riemannian manifolds with nonpositive sectional curvature have been of interest due to the rich interplay between their geometric, topological and dynamical properties and local-global properties like the Cartan-Hadamard theorem. In 1987, Gromov defined a notion of nonpositive curvature, *locally CAT(0)*, for the larger class of geodesic metric spaces. Locally CAT(0) spaces satisfy results that are analogues of results for nonpositively curved Riemannian manifolds. A natural question to ask is: how are these two notions of nonpositive curvature related? In my current research, I address the following question:

Question. *If M is a closed manifold with a locally CAT(0) metric, does it support a smooth Riemannian metric with nonpositive sectional curvature?*

In low dimensions, the answer is yes. In dim = 2, this follows from the classification of surfaces. In dim = 3, it is a consequence of Thurston's geometrization conjecture, which is now a theorem by Perelman and others. For dimensions ≥ 5, Davis and Januszkiewicz [4] showed that for each $n \geq 5$, there is a piecewise flat, nonpositively curved closed manifold M^n whose universal cover $\tilde{M}^n$ is not simply connected at infinity. In particular, M cannot have a smooth nonpositively curved Riemannian metric. In 2012, Davis, Januszkiewicz and Lafont [5] dealt with the remaining case of dimension 4. They construct a "knottedness" in the boundary at infinity of $\tilde{M}$ which gives the obstruction to nonpositively curved Riemannian smoothing. I extend the methods in [5] to provide new examples of locally CAT(0) 4-manifolds which do not support a non-positively curved

Riemannian metric. I show that "linking" in the boundary at infinity of the manifold is another obstruction and I prove the following:

Theorem ([10]). *There exists a 4-dimensional closed manifold M with the following properties:*

1. *M supports a locally CAT(0)-metric.*
2. *The boundary at infinity of its universal cover $\tilde{M}$ is homeomorphic to S^3, and $\tilde{M}$ is diffeomorphic to $\mathbb{R}^4$.*
3. *The maximal dimension of flats in $\tilde{M}$ is 2, and the boundary of every $\mathbb{Z}^2$-periodic 2-flat is a circle that is unknotted in $\partial^\infty \tilde{M}$.*
4. *$\pi_1(M)$ is not isomorphic to the fundamental group of any Riemannian manifold of non-positive sectional curvature.*

These examples are indeed different from those in [5], since their examples have $\mathbb{Z}^2$-periodic flats that are wild knots in the boundary, whereas in the examples that I give, all such flats are unknotted in the boundary. In particular, one can see that the obstruction comes from linking of the flats in the boundary.

The desired manifold is constructed using the *Davis complex* with a particular triangulation of S^3 as its nerve. For an n-component link L, let Δ be a smooth flag triangulation of S^3 with exactly n *isolated squares* - one square corresponding to each component of L. Construct a smooth 4-manifold, $M = P_\Delta$ so that $\tilde{M}$ is the Davis complex with nerve Δ. Then M is locally CAT(0) with boundary at infinity, $\partial^\infty \tilde{M}$, homeomorphic to S^3. For every vertex $v \in P_\Delta$, the *link at v, $lk_{P_\Delta}(v)$* is simplicially isomorphic to Δ. This means that the link at each vertex of P_Δ contains a copy of L. Further, there are n copies of the flat torus T^2 in M, each corresponding to a component of L. Suppose M' is a Riemannian manifold with nonpositive sectional curvature and $\pi_1(M') \cong \pi_1(M)$. Then there is an $\pi_1(M)$-equivariant homeomorphism $\tilde{\phi} : \partial^\infty \tilde{M} \to \partial^\infty \tilde{M}'$. The existence of such a homeomorphism is not true in general [3]. However, it is true if one of the spaces has *isolated flats* [8]. By Caprace [2], the isolated squares condition on Δ ensures isolated flats in $\tilde{M}$. Lifting to universal covers, the n tori lift to a collection of flats whose boundaries are homeomorphic to S^1, thus giving a link $L_\infty \hookrightarrow \partial^\infty \tilde{M}$. The link L_∞ is mapped to an isotopic link $L'_\infty \hookrightarrow \partial^\infty \tilde{M}'$ under the homeomorphism $\tilde{\phi}$. By the flat torus theorem, there exist flats F'_i in $\tilde{M}'$ such that $L'_\infty = (\partial^\infty F'_1, \ldots, \partial^\infty F'_n)$. If L is a non-trivial knot, then the local knottedness propagates to the boundary at infinity, and the embedding $\partial^\infty F \hookrightarrow \partial^\infty \tilde{M}$ defines a nontrivial knot, which leads to an obstruction to having a Riemannian metric with nonpositive sectional curvature [5]. The following result shows that if L is an unknot then we do not get the above obstruction.

Theorem ([10]). *Let k be an unknot, and Σ be a triangulation of S^3 of type k. Let M be the manifold as defined above. Let F be a flat in $\tilde{M}$ such that for every vertex $v \in F$, $lk_F(v)$ is an unknot in $lk_{\tilde{M}}(v)$. Then $\partial^\infty F$ is an unknot in $\partial^\infty \tilde{M}$.*

This leads us to explore the case where L has two or more unknotted components. We have the following corollary that generalizes Theorem 2 to links.

Corollary ([10]). *Let Σ be a triangulation of S^3 of type $L = (l_1, \ldots, l_m)$. Let M be the manifold as defined above. Let $\{F_1, F_2, \ldots, F_m\}$ be the collection of flats such that $lk_{F_i}(x)$ is a copy of the i-th component l_i, and further, the link given by $(lk_{F_1}(x), \ldots, lk_{F_m}(x))$ is isotopic to L. If each l_i is an unknot, then there is a homeomorphism of pairs $(\partial^\infty \tilde{M}, \amalg\ \partial^\infty F_i) \cong (lk_{\tilde{M}}(x), \amalg\ lk_{F_i}(x))$. In particular, $(\partial^\infty F_1, \ldots, \partial^\infty F_m)$ is isotopic to L.*

Assume that L is an n-component link that is not a great circle link. For a suitably chosen point $p \in \tilde{M}'$, the geodesic retraction $\partial^\infty \tilde{M}' \to T_p\tilde{M}'$ is a homeomorphism that maps the link L'_∞ to an isotopic link in the unit tangent sphere of p. Since, in a smooth Riemannian manifold the unit tangent spaces of flats are great circles. The *Whitehead link* and *Brunnian links* give a family of links, with linking numbers $= 0$, that generate such manifolds.

In higher dimensions. In order to extend this method for higher dimensions one will have to consider knotting and linking spheres of co-dimension 2. We still have a flat torus theorem but one might have to develop new tools to construct the appropriate triangulations of higher dimensional spheres. However, due to the work by Januszkiewicz and Świątkowski [9], one cannot get flag-no-squares triangulations for higher dimensions. I am currently exploring other techniques that can help us detect the obstruction to Riemannian smoothing within the manifold instead of the boundary at infinity.

Branched coverings. The first heorem relies on a basic rigidity phenomenon in nonpositive curvature, namely that a free abelian subgroup in the fundamental group of a closed nonpositively curved manifold stabilizes a flat in the universal cover of the manifold. Stadler [11] uses this rigidity to provide different examples of closed smooth 4-manifolds which support singular metrics of non-positive curvature but no smooth ones. In fact, the example was pointed out by Gromov in 1985 and it first appeared as Exercise 1 in [1]. The manifold is constructed as a branched covering of $\Sigma \times \Sigma$, with the branching locus being the diagonal Δ_Σ, where Σ is a higher genus surface. One notices that the diagonal $\Delta_\Sigma \hookrightarrow \Sigma \times \Sigma$ is a codimension 2 submanifold locally modelled on the inclusion $\mathbb{H}^2 \hookrightarrow \mathbb{H}^2 \times \mathbb{H}^2$. This is in contrast with the result by Gromov and Thurston [6], who proved that any finite ramified covering of a compact hyperbolic manifold, along a codimension 2 totally geodesic submanifold, can be endowed with a Riemannian metric of negative sectional curvature, provided the ramification locus has large normal neighborhood. The branching locus in the Gromov-Thurston examples are locally modelled on $\mathbb{H}^{n-2} \hookrightarrow \mathbb{H}^n$. I am currently exploring the following questions.

Problem. *When do branched coverings of non-positively curved manifolds over a co-dimension 2 submanifold as the branching locus admit a smooth Riemannian metric with non-positive sectional curvature?*

Problem. *Can one generalize Stadler's method to find examples in dimensions ≥ 5?*

2 Boundaries of Groups

We can rephrase Theorem 2 in terms of boundaries of CAT(0) groups. Let (W, S) be the right angled Coxeter group whose nerve $L = L(W, S)$ is the simplicial complex Δ, as constructed in the previous section. If the nerve L contains an unknotted square, then the boundary of the Davis complex $\Sigma(W, S)$ corresponding to the group (W, S) contains an unknotted square. One would like to generalize this to a general Coxeter group, and answer the question:

Problem. *If a graph K is embedded in the nerve $L = L(W, S)$ of a Coxeter group (W, S), does the boundary of its Davis complex $\Sigma(W, S)$ contain K as an embedding?*

In joint work with M. Haulmark and G.C. Hruska, I answer the above question for the special case of complete graphs K_n by proving the following.

Theorem ([7]). *Let W_n be the Coxeter group defined by a presentation with n generators of order two such that $(st)^3 = 1$ for all generators $s \neq t$. In other words, the nerve of W_n be the complete graph K_n with all the edges labeled 3.*

1. *The group W_3 has visual boundary homeomorphic to the circle and acts as a triangle reflection group on the Euclidean plane.*
2. *The group W_4 has visual boundary homeomorphic to the Sierpinski carpet and acts as a nonuniform arithmetic lattice on $\mathbb{H}^3$ commensurable with the fundamental group of the figure eight knot complement.*
3. *For each $n \geq 5$, the group W_n has visual boundary homeomorphic to the Menger curve.*

Haulmark has shown that if a one-ended CAT(0) group with isolated flats and one-dimensional boundary does not split over a virtually cyclic subgroup, then the boundary is either a circle, a Sierpinski carpet, or a Menger curve. By proving the above theorem, we provide a family of Coxeter groups that demonstrate all three possible outcomes of the CAT(0) boundary classification theorem. In particular, this gives the first explicit examples of groups with Menger curve boundaries, answering a question by K. Ruane. Our methods in proving the above result do not provide insight in answering the more general case of a given graph K. Hence the last problem still remains open and I am exploring it further.

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Big mapping class groups acting on simplicial things

Big mapping class groups are the mapping class groups of infinite-type surfaces; that is, surfaces with non-finitely-generated fundamental group. Since they're not finitely generated, we can't apply many of the traditional tools of GGT. But there are some nice simplicial graphs that they act on by isometries. By the time this is printed I should have uploaded a preprint proving that the mapping class group of a surface with some punctures is isomorphic to the automorphism group of that surface's *loop* or *relative arc graph* in the natural way. There's also a neat generalization of "finitely generated" to possibly uncountable topological groups, called "CB-generated". Briefly, a subset is *coarsely bounded* ("CB") if it is bounded in every left-invariant pseudo-metric, and the whole group is *CB-generated* if it is generated by a CB subset. The upshot is that "CB" can take the place of "finite" in a lot of standard GGT results. In previous years I've written here that I don't really know what I'm doing. That very much remains true, all of the above notwithstanding.

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What I'm interested in

The common denominator of my various research interests is the study of invariant functionals on function spaces of (possibly topological) groups, or on spaces that admit a group action. This includes the notion of amenability and the related, stronger property of superamenability (see [1]). Alternatively, invariant functionals are intimately related to fixed-point properties for groups, as they are in effect fixed points in the dual of some vector space. This point of view drives my interest in fixed-point results for groups in general (see [1]). I'm also interested in invariant measure. Because when we are speaking about invariant functionals on locally compact spaces we are actually speaking about measure on it, thanks to the Riesz Representation Theorem. In these terms, I am interested in understanding when a group action on a locally compact space fixes a (Radon) measure (see [3]). I am also really fascinated by the theory of representation of locally compact groups. This includes harmonic analysis on such groups, the theory of induced representations, unitary representations and Gelfand pairs.

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Word maps and word measures

I have just started a PhD with Doron Puder as my advisor at Tel-Aviv University. The topics I intend to research are word maps and word measures, both generally and specifically in unitary groups. Let me make a brief introduction to word maps: Fix a word w in the free group F_k. For every group G, the word w induces a map $G^k \to G$ by substitution. For example, the commutator word $w = [a, b] \in F_2$ defines a map $G \times G \to G$ by mapping $(x, y) \mapsto [x, y]$. Many natural questions arise in this context in the attempt to relate the algebraic properties of the word w to the properties of the map it induces. If the group G is finite or compact, it is naturally equipped with a uniform/Haar probability measure on G^k, and the word map induced by w yields a pushforward measure on G which is called the *word measure*. An example of an open question in this area is the following (see [10] and references within):

Conjecture (Amit-Vishne, Shalev)**.** *Two words $w_1, w_2 \in F_k$ are in the same orbit of the automorphism group of F_k iff the word measures they induce on any finite group G are equal.*

I did my master's degree at the Hebrew University in probability theory with Ori Gurel-Gurevich as my advisor. My thesis was in the topic of random walks on circle packings. The motivation for looking at circle packings comes from many different areas of mathematics, such as complex analysis (see Rodin and Sullivan's paper [7]), discrete complex analysis and probability theory. A *circle packing* is a collection of circles in the plane with disjoint interiors. The *tangency graph* of a circle packing is the graph obtained from it by assigning a vertex to each circle and connecting two vertices by an edge if their respective circles are tangent. The celebrated Koebe-Thurston-Andreev Circle Packing Theorem [1, 2] states that every finite planar graph is isomorphic to the tangency graph of some circle packing. Furthermore, if the graph is a triangulation then the circle packing representing it is unique up to Möbius transformations and reflections across lines in the plane. A concise background on the probabilistic and combinatorial properties of circle packings can be found in [3]. Now consider an infinite planar triangulation G. A compactness argument shows that, as in the finite case, G can be circle packed. However, the question of uniqueness is more complicated. We define the *carrier* of an circle packing of an infinite triangulation to be the union of all the circles, their interiors and the spaces bounded between three mutually tangent circles (interspaces). It turns out that there exists an infinite planar triangulation that can be circle packed in a first way such that the carrier is the open unit disc and in a second way such that the carrier the open unit square, but a Möbius tranformation cannot map the unit disc to the unit square. In [5, 6], He and Schramm extended the theory of circle packings to the infinite case. They proved the following remarkable theorem, relating circle packings to the probabilistic property of recurrence: Let G be a one-ended planar triangulation with bounded degrees. Then exactly one of two cases applies to G: either it can be circle packed with the entire plane as carrier and the simple random walk on it is recurrent, or it can be circle packed with the open unit disc as carrier and the simple random walk on it is transient. In [4], Gurel-Gurevich, Nachmias and Souto extended the He-Schramm theorem to the multiply-ended case, showing that a planar triangulation with bounded degrees can be circle packed with a parabolic carrier iff the simple random walk on it is recurrent (a domain $\Omega \subseteq \mathbb{R}^2$ is called *parabolic* if for any open set $U \subseteq \Omega$, Brownian motion started at any point of Ω and killed at $\partial\Omega$ hits U almost surely). My thesis dealt with the extension of the He-Schramm theorem to the case of planar triangulations of unbounded degree. Generally, the theorem can fail in such a case: there exists a circle packing of a (unbounded-degree) transient planar triangulation with the entire plane as carrier. A possible solution which can make the theorem still hold is replacing the simple random walk with a weighted one, with edge weights induced by the geometry of the circle packing. These weights arise naturally in the context of discrete complex analysis [9], and were proposed in this context by Dubejko [8]. The weights have a few nice properties: First, in the bounded-degree case, the weighted random walk is recurrent iff the simple random walk is recurrent: thus, one can replace the simple walk with the weighted one in the statement of the He-Schramm theorem. Second, the sequence of centers of circles visited in the weighted random walk is a martingale. In my thesis, I showed that given a circle packing of an infinite planar triangulation with a parabolic carrier, the weighted random walk on it is recurrent.

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About unitary representations of locally compact groups

I just finished my studies and started my Ph.D the 1st of October of last year. Therefore, my previous research work is almost entirely limited to my master thesis. This one aimed at providing an introduction to the general theory of unitary representations of locally compact groups but also contained a detailed and complete exposition of an advanced theorem originally due to Ol'Shianskii (see [1]) and studied in detailed in the book of Alessandro Figà-Talamanca and Claudio Nebbia [2], that classifies all irreducible unitary representations of a specific family of non compact locally compact groups, namely the full automorphism group of regular locally finite trees. In the continuity of this master thesis, my Ph.D. research project aims at studying the relation between the algebraic structure of a locally compact group and the properties of its (possibly infinite-dimensional) continuous unitary representations. Building upon the recent progress in the structure theory of locally compact groups, my research project will hopefully contribute to the Type I Conjecture on groups acting on trees, the spectral decomposition of Koopman representations associated with tree lattices, and the characterization of the locally compact groups that are irreducibly faithful. In particular, following the work of Olivier Amann [3], who successfully generalised the classification given in [2] to closed subgroups of $Aut(T)$ satisfying the Tits independence IP_0 (see [4]), I'm currently trying to adapt his arguments to some classification of unitary representations of closed subgroups of $Aut(T)$ satisfying a weaker independence IP_k for some $k > 0$. The obtention of this classification being motivated by the fact that every closed subgroup $H \leq Aut(T)$ is the intersection of a descending chain of closed subgroups, each satisfying IP_k for values of k that tend to infinity (see [4]).

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Accessibility of Groups

I am interested in Coxeter groups, in particular trying to understand group-combinatorial properties using the geometry of the action of the group on its Coxeter complex. I am also curious about questions relating to the accessibility of groups and understanding other finiteness properties using tools like Bestvina-Brady Morse theory. Given a Coxeter group W together with a set of generators S, there is an important partial order on W called the Bruhat order in which $u \leq v$ if some reduced word for v contains a subword which is a reduced word for u. If we write $[1, v] := \{u \in W \mid u \leq v\}$, this is finite and graded by the length function l on W. This allows us to define the Poincaré polynomial of v to be $P_v(q) := \sum_{u\in[1,v]} q^{l(u)}$. One question in which I have been very interested is when can one characterise whether $P_v(q)$ is palindromic (in the sense that the coefficients read the same forwards and backwards) using geometric properties of the Coxeter complex of W. This question is motivated by the work of S. Oh and H. Yoo who showed that when W is an irreducible Weyl group, $P_v(q)$ is palindromic if and only if $P_v(q) = R_v(q)$, where $R_v(q)$ counts the regions in the inversion hyperplane arrangement of v in the Coxeter complex of W [2]. In this case, this tells us about the smoothness of Schubert varieties in the flag manifold associated to W. Their result does not generalise to infinite Coxeter groups, however the geometry of the Coxeter complex can be used to partially factorise $P_u(q)$, a step which is essential to their work, making it easier to check palindromicity. Recently I have been thinking about accessibility questions of groups. These ask whether the process of starting with a f.g. group G, splitting it as a graph of groups, then passing to the vertex groups and repeating, terminates. One needs to place assumptions on the group G and the types of edge groups allowed. For example M.J. Dunwoody showed that if G is (almost) f.p. and all edge groups are finite, then G is accessible [1]. Conversely there are examples of f.g. groups which are not accessible over finite edge groups.

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Leighton's Theorem and cube complexes

Leighton's Theorem states that any two finite graphs with a common universal cover have a common finite cover [2]. I am interested in generalisations of this theorem, and ways that they can be applied to rigidity problems.

In [3], joint with Gardam and Woodhouse, we proved a generalisation of Leighton's Theorem that gives us control over the local structure of the common finite cover. More precisely, if T is the common universal cover and the deck transformation groups of the covers of the two finite graphs are $\Gamma_1, \Gamma_2 < \mathrm{Aut}(T)$, then a common finite cover corresponds to finding a $g \in \mathrm{Aut}(T)$ such that Γ_1 and $g\Gamma_2 g^{-1}$ are commensurable in $\mathrm{Aut}(T)$; what we showed is that, for any integer R, one can choose this g such that its restriction to any R-ball in T is equal to an element of $\langle \Gamma_1, \Gamma_2 \rangle$. In [4] Woodhouse proved a similar generalisation in which the graphs are endowed with the extra structure of fins, which he used to exhibit quasi-isometric rigidity of certain amalgams of free groups over $\mathbb{Z}$. Woodhouse and I hope to extend this result to more general graphs of free groups.

Another way one might try to generalise Leighton's Theorem is to replace graphs with non-positively curved cube complexes. In general such a theorem cannot hold, because for instance Burger-Mozes constructed square complexes with simple fundamental group and universal cover a product of regular trees [1]. However, cube complex Leighton's Theorem might hold if we restrict to virtually special cube complexes; this has a better chance of being true because special cube complexes have lots of finite covers - more precisely, they have residually finite fundamental groups, and subgroups induced by locally convex subcomplexes are separable. Nevertheless this is an open problem, indeed it is even unknown whether Leighton's Theorem holds for virtually special $\mathcal{VH}$-square-complexes.

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Finite rigidity Problems

I study simplicial complexes associated to surfaces and how the automorphism groups of these simplicial complexes relate to the mapping class group of their associated surfaces. One such complex which has been of particular interest to me is the *arc complex*, $\mathcal{A}(S)$, of a surface S with marked points. The vertices of $\mathcal{A}(S)$ correspond to isotopy classes of essential arcs on S, and the k-simplices correspond to collections of $k+1$ distinct classes of arcs with pairwise disjoint representatives.

A homeomorphism $H : S \to S$ induces an automorphism $H_* : \mathcal{A}(S) \to \mathcal{A}(S)$. Conversely, Irmak–McCarthy proved in [1] that any automorphism $\varphi : \mathcal{A}(S) \to \mathcal{A}(S)$ is induced by a homeomorphism of S, unique up to homotopy in most cases. In these cases $\mathrm{Aut}(\mathcal{A}(S)) \cong \mathrm{Mod}^{\pm}(S)$, a result referred to as the *rigidity* of $\mathcal{A}(S)$.

I recently proved the following. Suppose S is a compact, connected, orientable, finite-type surface with marked points, not the sphere with three marked points. Then there exists a finite simplicial subcomplex $\mathcal{X}$ of $\mathcal{A}(S)$ with the following property: If S' is a surface with $\dim(\mathcal{A}(S)) = \dim(\mathcal{A}(S'))$ and $\lambda : \mathcal{X} \to \mathcal{A}(S')$ a locally injective simplicial map, then there is a homeomorphism $H : S \to S'$ which induces λ, unique up to homotopy in most cases. The set $\mathcal{X}$ is referred to as a *finite rigid set* of $\mathcal{A}(S)$.

Currently, I am thinking about how to find finite rigid sets in other complexes. I am also considering how much of my theorem could be retained if I were to remove or loosen conditions.

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Co-volume of lattices in Lie groups controls their complexity

Consider S, a genus $g \geq 2$ surface. One can put a hyperbolic metric on it. Using the Gauss-Bonnet Theorem, we get

$$\int_S K = (2\pi - 2g)$$

But the curvature is identically -1, so on the left hand side we get the volume. In other words, if we know the volume of a surface, we know it's genus. But in dimension 2 this determines the topology of the surface completely! In my research, I try to generalize this phenomenon in various directions. In this case, we actually considered a lattice $\Gamma \leq SL_2(\mathbb{R})$, where Γ is the fundamental group of S. More generally we connect the volume of higher dimensional spaces of this type (locally symmetric spaces) to their complexity, be it topological or algebraic, for example with results of the type:
Let $\Gamma \leq G$ be a lattice in a semi-simple Lie group with no compact factors, then there exists a constant $C = C(G)$ such that

$$d(\gamma) \leq C \operatorname{vol}(G/\Gamma)$$

Where $d(\gamma)$ denotes the minimal number of generators of Γ.

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Actions of Artin groups on CAT(0) spaces and the $K(\pi, 1)$ conjecture

After doing my master thesis on actions of Artin groups on systolic spaces in Heidelberg with Petra Schwer (now Magdeburg, Germany), I start my PhD in Dijon (France) in October under the supervision of Luis Paris and Thomas Haettel (Montpellier) on actions of Artin groups on CAT(0) spaces and the $K(\pi, 1)$ conjecture. Artin groups were introduces by Tits [1] in 1966 as an extension of Coxeter groups, but they only gained their reputation later through a series of articles by Brieskorn, Saito and Deligne in relation to some algebraic varieties. There are only few results on all Artin groups and questions like the existence of torsion or a solution to the word problem are still open. As a matter of fact, the theory mostly consists in the study of more or less vast families. The two most studied and best understood families are the groups of spherical type and right-angled Artin groups. The study of Artin groups, whether globally or through a chosen family, is a rapidly growing subject which abounds with open questions. The $K(\pi, 1)$ conjecture and the existence of proper actions on CAT(0) spaces are two iconic question on this subject. My research project is based on those two questions. As I am currently at the very beginning of my PhD, I devote most of my time to reading articles on the topic. At the same time and as a continuation of my master thesis, I try to construct systolic spaces (a combinatorial version of CAT(0) spaces introduces in [2]) on which braid groups could act. The existence of cocompact, free and properly discontinuous actions of braid groups on CAT(0) spaces is a fundamental problem of this topic.

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Relations between the Bogomolov property and the property $(\hat{\tau})$ of groups

On the ICM 2014 in Seoul Emmanuel Breuillard pointed out a question of Jordan Ellenberg from 2012 (Section 4.10 [3], Question 14 [4]): An algebraic extension $K/\mathbb{Q}$ is said to have the Bogomolov property if an $\epsilon > 0$ exists such that for the height on K and for all $\alpha \in K$ it either holds $h(\alpha) = 0$ or $h(\alpha) > \epsilon$. The height of $\alpha \in K\backslash\{0\}$ is defined as

$$h(\alpha) = \sum_{v \in M_K} \max\left(0, \log|\alpha|_v^{\mu(v)}\right)$$

where M_K is the set of all absolute values on K and $\mu(v) := \frac{[K_v:\mathbb{Q}_p]}{[K:\mathbb{Q}]}$ is the local degree of $v \in M_K$, for $v|p$ and $p \in M_{\mathbb{Q}}$, divided by the degree of $K/\mathbb{Q}$. The set $M_{\mathbb{Q}}$ for example consists of all p-adic absolute values and the archimedean absolute value $|.|_\infty$. Furthermore we set $h(\alpha) = 0$. The height measures the arithmetic complexity of an algebraic number. The rational numbers $\mathbb{Q}$ for example have the Bogomolov property since $h\left(\frac{a}{b}\right) = \log\left(\max\left(|a|, |b|\right)\right) \geq \log(2)$ or $= 0$ for $a \in \mathbb{Z}\backslash\{0\}$, $b \in \mathbb{N}\backslash\{0\}$ and a, b coprime. Let $\mathbb{Q}^{\mathrm{ab}}$ be the maximal Galois extension of $\mathbb{Q}$ whose Galois group is abelian, and $K/\mathbb{Q}$ a possibly infinite Galois extension. We say that a profinite group G (e.g. $G = \mathrm{Gal}(K\mathbb{Q}^{\mathrm{ab}}/\mathbb{Q}^{\mathrm{ab}})$) has Ellenberg's property $(\hat{\tau})$ if G is topologically finitely generated by a finite subset S and for any such S there is an $\epsilon_S > 0$ such that the following holds: Let N be a finite-index normal subgroup of G and S_N the image of S under the quotient map $G \twoheadrightarrow G/N$. Let $\Gamma(G/N, S_N)$ be the Cayley graph of G/N with respect to the finite generating set S_N. Then the Cheeger constant of this Cayley graph has to be greater than ϵ_S. The Cheeger constant of a finite simple graph Γ is defined as

$$h(\Gamma) := \min_{A \subseteq V(\Gamma)} \frac{|\partial A|}{\min\{|A|, |V(\Gamma)\backslash A|\}}.$$

where $V(\Gamma)$ denotes the set of vertices of Γ, and ∂A the set of edges between A and its complement $V(\Gamma)\backslash A$. Does K have the Bogomolov property if $G = \mathrm{Gal}(K\mathbb{Q}^{\mathrm{ab}}/\mathbb{Q}^{\mathrm{ab}})$ has Ellenberg's property $(\hat{\tau})$? The aim of my PhD project, which I started in April 2018, is to study this question of Jordan Ellenberg. Note that the implication is already known if K is a finite Galois extension of $\mathbb{Q}^{\mathrm{ab}}$: All finite groups have Ellenberg's property $(\hat{\tau})$ and all finite algebraic extensions of $\mathbb{Q}^{\mathrm{ab}}$ have the Bogomolov property (see [1]). The above problem has similarities to another interplay between diophantine geometry and geometric group theory, the equivalence of the Lehmer conjecture and the growth conjecture for $d = 2$ (Theorem 6.4.10 [5]). In 1933 Derrick Henry Lehmer formulated the still unsolved conjecture that there exists a real number $\epsilon > 0$ such that $h(\alpha) \cdot \deg(\alpha) > \epsilon$ holds for all algebraic numbers α apart from zero and roots of unity. The lowest known number for $h(\alpha) \cdot \deg(\alpha)$ is approximately $\log(1.17628)$, which holds for any root α of the polynomial $x^{10} + x^9 - x^7 - x^6 - x^5 - x^4 - x^3 + x + 1$ and was already found by Lehmer almost hundred years ago. Not even large-scale calculations with modern computers have found a lower one so far. The growth conjecture says that for every $d \in \mathbb{N}\backslash\{0\}$ there exists a real number $\epsilon_d > 0$ with the following property: For every field K and every finite set $S \subseteq \mathrm{GL}(d, K)$ either $\varrho_{\langle S\rangle_{\mathrm{GL}(d,K)},S} = 1$ and $\langle S\rangle_{\mathrm{GL}(d,K)}$ is virtually nilpotent, or $\varrho_{\langle S\rangle_{\mathrm{GL}(d,K)},S} > 1 + \epsilon_d$ (Conjecture 6.4.5 [5]). Here $\varrho_{\langle S\rangle_{\mathrm{GL}(d,K)},S}$ denotes the exponential growth rate of $\langle S\rangle_{\mathrm{GL}(d,K)}$ with respect to S. Emmanuel Breuillard proved the above-mentioned equivalence by relating the Mahler measure of an algebraic number α to the growth rate $\varrho_{\langle S_\alpha\rangle_{\mathrm{GL}(2,\overline{\mathbb{Q}})},S_\alpha}$, where

$$S_\alpha := \left\{ \begin{pmatrix} \alpha & 0 \\ 0 & 1 \end{pmatrix}^{\pm 1}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{\pm 1}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\},$$

in the following way: $\log\left(\varrho_{\langle S_\alpha\rangle_{\mathrm{GL}(2,\overline{\mathbb{Q}})},S_\alpha}\right) \leq h(\alpha)\cdot\deg(\alpha)$ One of my results so far is the positive proof of Ellenberg's question in the case of a virtually solvable extension $K/\mathbb{Q}$, which means that $\mathrm{Gal}(K/\mathbb{Q})$ is virtually solvable. A group is called virtually solvable if it has a subgroup of finite index that is solvable. The supervisors of my doctoral thesis are Walter Gubler, who is an expert of heights in diophantine geometry (see [2]), and Clara Löh, who is an expert in geometric group theory (see [5]). Furthermore, in October 2019 I was invited by Emmanuel Breuillard to the University of Cambridge in order to work together on Ellenberg's question. My research interests in geometric group theory are the growth of groups, (non-)

amenable groups, expander graphs, superstrong approximation of groups etc. Above all I am searching after infinite Galois groups over $\mathbb{Q}^{ab}$ having Ellenberg's property $(\hat{\tau})$.

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Random walks on Cayley and Schreier graphs

Throughout my PhD, I have explored several somewhat varying questions. My first topic was a specific group. In his article [1], Monod defines a class of groups $H(A)$ of piecewise projective homeomorphisms on the real line, depending on a subring A of $\mathbb{R}$. They never have a free subgroup, and if A is dense Monod proves them to be non-amenable by comparing the orbit equivalence relation with that of $PSL_2(A)$. I studied $H(\mathbb{Z})$, the amenability of which is an open question. It is known that a group is non-amenable if and only if every non-degenerate measure on it has non-trivial Poisson boundary. I have obtained in [2] the non-triviality of the Poisson boundary for certain classes of measures. In particular, I have shown that for a finitely generated subgroup of $H(\mathbb{Z})$, either it is solvable or any strictly non-degenerate measure on it with finite first moment has non-trivial Poisson boundary. Further developing from that, I have also shown in [3] similar results for induced random walks on Schreier graphs. In particular, those results extend (slightly) what was previously known about random walks on Thompson's group F (which is a subgroup of $H(\mathbb{Z})$).

Afterwards, I worked on a different subject - studying the exact values of Følner functions for a given group and generating set. I have obtained results for the lamplighter group and $BS(1,2)$ and hope to extend it to $BS(1,n)$, but they seem resistant to real generalisations. Currently, I have begun work on transient graphs that have no transient subtrees (for the simple random walk).

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Left orders and formal languages

I am interested in studying the complexity of left-orders on finitely generated groups through the lens of formal languages and the Chomsky hierarchy. Left-orders on a group G are in bijections with semigroups P (called positive cones of left orders) with the property that P, P^{-1} and $\{1\}$ form a partition of G. Some examples of groups with positive cone are $\mathbb{Z} = \langle a \rangle$ with positive cone $\langle a \rangle^+$ and the Klein bottle group $K_2 = \langle a, b \mid bab = a \rangle$ with positive cone $\langle a, b \rangle^+$. These are examples of groups with finitely generated positive cones, a special case of positive cones that can be described by a regular language. Examples of groups which do not admit a finitely generated positive cone are $\mathbb{Z}^2$ (a finite index subgroup of K_2), and F_2. In fact, $\mathbb{Z}^2$ admit positive cones which can be described by regular language, whereas F_2's positive cone must be described at the minimum by context-free languages [1]. My first preprint [3] addressed the following questions. First, the example of $\mathbb{Z}^2 \leq K_2$ suggests that being finitely generated as a positive cone is a fragile property as it does not pass to finite index subgroup. We showed that regularity, the lowest complexity in the Chomsky hierarchy, does however pass to finite index. Second, Navas [2] showed that there is an infinite family of groups with 2-generated positive cones, which addressed the fact that not many examples of groups with finitely generated positive cones are known. We resolved the next natural question, which is finding for every $k \geq 3$ an infinite family of groups with of k-generated positive cones. As a corollary, we get that $F_2 \times \mathbb{Z}$ is finitely generated. This contrasts the result of [1] which states that positive cones of free products are at minimum context free. Finally, I generalized this result of [1] to acylindrically hyperbolic groups. I am currently working on more problems with a similar flavour. RAAGs are known to be left-orderable, but no explicit left-order which works for all of them is known. I would like to find a context-free order for all RAAGs.

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Acylindrically hyperbolic groups

I am primarily interested in the theory of acylindrically hyperbolic groups. The class of acylindrically hyperbolic groups was introduced by Osin as a generalization of non-elementary hyperbolic and non-elementary relatively hyperbolic groups. By definition, a group is *acylindrically hyperbolic* if it admits a non-elementary acylindrical action on some Gromov hyperbolic space by isometries. Examples of acylindrically hyperbolic groups can be found in various classes of groups that interest group theorist for years: mapping class groups of punctured closed orientable surfaces, outer automorphism groups of finite rank free groups, small cancellation groups, 3-manifold groups, the Cremona group, non-elementary convergence groups, etc. Moreover, acylindrically hyperbolic groups share strong algebraic, analytic, and geometric properties. First, every acylindrically hyperbolic group G satisfies $H_b^2(G, \ell^2(G)) \neq 0$, which opens the door for applications of the Monod-Shalom rigidity theory to G. Second, acylindrically hyperbolic groups satisfy an algebraic analog of Thurston's hyperbolic Dehn filling theorem, which can be used to prove useful algebraic properties (eg. SQ-universality). Third, the classical small cancellation theory can be generalized to the context of acylindrically hyperbolic groups. For other examples and properties of this class of groups, we refer to the survey [1]. Over the past several years, I have worked on two projects about acylindrically hyperbolic groups. Below are brief descriptions of them.

A dynamical characterization of acylindrically hyperbolic groups

This project is motivated by the close relationship between convergence groups and acylindrically hyperbolic groups. The notion of a convergence group was introduced by Gehring and Martin in order to study Kleinian groups via their actions on the ideal sphere of $\mathbb{H}^3$. Let G be a group acting on a compact metrizable topological space M. Then the action is called a *convergence action* (or G is called a *convergence group*) if the induced diagonal action of G on the configuration space

$$\Theta_3(M) = \{(x, y, z) \mid x \neq y, y \neq z, z \neq x\}$$

is properly discontinuous, where $\Theta_3(M)$ endows with the subspace topology inherited from the product topology of M^3. Examples of convergence groups include groups acting properly discontinuously on proper Gromov hyperbolic spaces, in particular hyperbolic and relatively hyperbolic groups. If G is a convergence group acting on a topological space M, then G is called *non-elementary* if it does not fix set-wise a non-empty subset of M with at most two points. Non-elementary convergence groups and acylindrically hyperbolic groups share many properties. For instance, if a group G is either non-elementary convergence or acylindrically hyperbolic, then (1) G is not invariably generated, (2) G admits a primitive action with finite kernel, and (3) the reduced C^*-algebra of G is simple provided that G has no non-trivial finite normal subgroup. This phenomenon suggests that one of these two classes of groups is contained in the other, or these two classes coincide. By adapting a construction of Bowditch, we proved that non-elementary convergence groups are acylindrically hyperbolic [2]. The rich theory of acylindrically hyperbolic groups can then be applied to non-elementary convergence groups. In general, an acylindrically hyperbolic group is not necessarily a non-elementary convergence group. Examples can be found in mapping class groups. We generalized the convergence property to a weaker dynamical property which characterizes acylindrical hyperbolicity.

Cohomology of group theoretic Dehn fillings

In 3-dimensional topology, Dehn filling is the process of gluing a solid torus to a 3-manifold M with toral boundary by identifying their boundaries. Topologically distinct ways of gluing a solid torus are parametrized by free homotopy classes of essential simple closed curves on ∂M, called slopes. Thurston's hyperbolic Dehn filling theorem asserts that if M is a 3-manifold such that ∂M is a torus and $M \setminus \partial M$ admits a complete finite volume hyperbolic structure, then for all but finitely many slopes s on ∂M, M_s, the Dehn filling of M corresponding to s, is a hyperbolic manifold. There is an algebraic analog of Dehn filling, called group theoretic Dehn filling. Given a group G, its subgroup H, and a normal subgroup N of H, the group theoretic Dehn filling associated with these data is the quotient $G/\langle\langle N \rangle\rangle$, where $\langle\langle N \rangle\rangle$ is the normal closure of N in G. The relationship between the two versions of Dehn filling can be seen via the following example: Under the assumptions of Thurston's hyperbolic Dehn filling theorem, $\pi_1(\partial M)$ is naturally a subgroup of $\pi_1(M)$, and every slope s generates a normal subgroup $N_s = \langle s \rangle$ of $\pi_1(\partial M)$. We have $\pi_1(M_s) = \pi_1(M)/\langle\langle N_s \rangle\rangle$ by the Seifert-van Kampen theorem. Acylindrically hyperbolic groups exhibit nice properties under group theoretic Dehn fillings. For instance, an algebraic analog of Thurston's hyperbolic Dehn filling theorem was proved for acylindrically hyperbolic groups by Dahmani, Guirardel, and Osin. In [3] and subsequent works, we obtained a spectral sequence to compute cohomology of Dehn filling quotients of acylindrically hyperbolic groups. As an application, we improved known results on acylindrically hyperbolic groups by adding cohomological finiteness conditions. For a group G, denote the cohomological dimension of G by $cd(G)$. We showed that if G is acylindrically hyperbolic and C is any countable group, then C embeds into an acylindrically hyperbolic quotient Q of G such that $cd(Q) \leqslant \max\{cd(G), cd(C)\}$. We also showed that if G_1 and G_2 are finitely generated acylindrically hyperbolic groups, then there is a common acylindrically hyperbolic quotient G of G_1 and G_2 such that $cd(G) \leqslant \max\{cd(G_1), cd(G_2)\}$.

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Amenability of topological full groups

In my PhD thesis I investigate the extensive amenability of group actions and amenability of certain topological full groups. Let G be a group acting on a compact space X by homeomorphisms. The *topological full group* $[[G \curvearrowright X]]$ of this action is the group of all homeomorphisms of X which are piecewise given by elements of G, such that each piece is open in X. These groups are widely studied, especially in the case when $X = C$ is a Cantor space, and $G = \mathbf{Z}$ is the group of integers. Matui ([3]) proved that the commutator subgroup of $[[\mathbf{Z} \curvearrowright C]]$ is simple for any minimal action $\mathbf{Z} \curvearrowright C$. He also showed that this simple group is finitely generated in many cases. Extensive amenability of group actions was introduced and first used in [2], the definition and the name were given in [1]. With the help of this tool Juschenko and Monod proved that the topological full group of any minimal action $\mathbf{Z} \curvearrowright C$ is amenable ([2]). Combining this with the results of Matui gives the first examples of finitely generated amenable simple groups. My goal was to answer the following question: If $G \curvearrowright X$ is a minimal action, when does it follow that $[[G \curvearrowright X]]$ is amenable? I managed to establish a result for all finitely generated groups ([4]):

Theorem. *Let G be a finitely generated group.*

- *If G is virtually cyclic, then for any compact space X, and any minimal action $G \curvearrowright X$, the topological full group $[[G \curvearrowright X]]$ is amenable.*
- *If G is not virtually cyclic, then there exists a free, minimal action of G on a Cantor space C, such that $F_2 \leq [[G \curvearrowright C]]$, i.e., the topological full group contains a non-abelian free group.*

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Tits Alternative for 2-Dimensional Artin Groups

We say a group G satisfies the *Tits Alternative* if for every finitely generated subgroup H, H contains a free group of rank 2 or H is virtually solvable. My research deals with investigating if a certain class of Artin groups satisifes the Tits Alternative. We say a group is an *Artin group* if it has the following presentation:

$$A_{s_1,...,s_n} = \langle s_1, ..., s_n | \forall i \neq j, s_i s_j s_i ... = s_j s_i s_j ..\rangle$$

where both sides of the equality have the same number of letters, denoted m_{ij}. To A, we associate its corresponding *Coxeter group* by adding in the relation that all generators have order 2. Thus, we associate to A the group

$$W_{s_1,...,s_n} = \langle s_1, ..., s_n | \forall i, s_i^2 = 1; \forall i \neq j, s_i s_j s_i ... = s_j s_i s_j\rangle$$

We say A is *two dimensional*, if for any three generators $\{r, s, t\} \subset \{s_1, ..., s_n\}$, $W_{r,s,t}$ is infinite. We can rephrase the condition of two dimensionality as

$$\frac{1}{m_{rs}} + \frac{1}{m_{st}} + \frac{1}{m_{rt}} \leq 1$$

Let us remark on some partial results: in the case where we have strict inequality, then Tits Alternative is true (see [1]). In the case all generators have $m_{ij} \geq 3$, Tit's Alternative is true (see appendix of [2]). My research focuses on the general case.

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Right-angled Artin groups, CAT(0) cube complexes and mapping class groups.

Let Γ be a simplicial graph with vertex set $V(\Gamma) = \{v_1, ..., v_n\}$. The *right-angled Artin group* (Raag) associated to Γ, noted $A(\Gamma)$, is the group with presentation

$$A(\Gamma) = \langle v_1, \cdots, v_n \mid [v_i, v_j] \text{ for all } \{v_i, v_j\} \in E(\Gamma)\rangle$$

Raag's therefore interpolate between free abelian groups and free groups. On the one hand, Raag's are interesting to study because, despite looking fairly simple at a first glance, they posses many complex features. Therefore, I wish to study them further, especially by using CAT(0) geometry. The main idea is as follows. The Salvetti complex of $A(\Gamma)$, noted $S(A(\Gamma))$ is a cube complex whose 2-skeleton is the presentation complex of $A(\Gamma)$ and whose fundamental group is $A(\Gamma)$. Moreover, it turns out that $S(A(\Gamma))$ is a locally CAT(0) space. Daniel T. Wise studied CAT(0) cube complexes from a combinatorial point of view, heavily relying on the structure of some subspaces called hyperplanes. This theory has a beautiful interplay with the study of 3-manifolds and I wish to keep exploring this approach. On the other hand, since Raag's have been proven to already have many nice properties, one can study other groups by finding Raag's subgroups. I am particularly interested in this point of view for the study of the mapping class group of surfaces. In a series of articles, Kim and Koberda introduce and study the *extension graph* Γ^e of a $A(\Gamma)$. This graph has as a vertex set all the conjugate of the generators of $A(\Gamma)$ and two vertices are joined by an edge if corresponding elements commute in $A(\Gamma)$. Kim and Koberda proved that every RAAG embeds in the mapping class group of some closed surface Σ as a subgroup generated by powers of Dehn twists. They also show that at the same time, the extension graph Γ^e embeds into the curve graph of $C(\Sigma)$. Moreover, Γ^e and $C(\Sigma)$ share enough properties that it seems natural to thing of them as anolog to each other. One of my current interest is to study the relation between Γ^e and $C(\Sigma)$ further. By doing so, I hope not only to understand the structure of RAAG's better but also the subgroup structure of mapping class groups. On top of that, there are still a lot of open question regarding the structure of the automorphism groups of right-angled Artin groups and I plan to investigate this in the future. Finally, on the side, I also have interest in other subjects such as abstract polytopes, Fushcian groups, Kleinian groups, tesselations, ...

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Random triangular groups at density 1/3

Let $\langle S|R\rangle$ be a group presentation with a set of generators S and a set of relations R. Consider the following Bernoulli model $\Gamma(n,p)$ of a random group : S consists of n generators, while R consists of relations taken independently at random with probability $p = p(n)$ among all cyclically reduced words of length three. Note that there are around $8n^3$ relations in total. Groups induced by presentations obtained in this way are called triangular groups. This model is introduced in [1], a little bit different to the absolute triangular model in [2]. We are interested in the asymptotic behaviour of the random triangular group $\Gamma(n,p)$ as $n \to \infty$. For a given function $p = p(n)$ we say that $\Gamma(n,p)$ has a given property a.a.s. (asymptotically almost surely) if the probability of $\Gamma(n,p)$ having this property tends to 1 as $n \to \infty$.
The lemma 12 in [1] states that if the probability of taking relations is low enough, say $p \leq \log n/(25n^2)$, then a.a.s. there exists a generator $s \in S$ such that neither s nor s^{-1} belongs to any relation in $\Gamma(n,p) = \langle S, R\rangle$. Which means that the group $\Gamma(n,p)$ can be decomposed into a free product $G * \mathbb{Z}$. The group is said to be at density 1/3 in this situation because around $pn \sim n \log n$ relations are taken, and $\log(pn)/\log(8n^3) \sim 1/3$ while $n \to \infty$. With the same idea, we can actually take a pack of generators instead of one. More precisely, we have the

Theorem. *For all $0 \leq \varepsilon < 1$, if $p \leq (1-\varepsilon)\log n/(50n^2)$ then the group $\Gamma(n,p)$ has a decomposition $G * \mathcal{F}(n^\varepsilon)$ where $\mathcal{F}(n^\varepsilon)$ is a free group of rank n^ϵ.*

Proof. Let X be the random variable which denote the number of generators not appearing in R. It suffice to show that a.a.s. $X \geq n^\varepsilon$. First we have

$$\mathbb{P}(X < n^\varepsilon) = \mathbb{P}(\mathbb{E}(X) - X > \mathbb{E}(X) - n^\varepsilon)$$

$$\leq \mathbb{P}(|\mathbb{E}(X) - X| \geq \mathbb{E}(X) - n^\varepsilon)$$

$$\leq \frac{\mathrm{Var}(X)}{(\mathbb{E}(X) - n^{\varepsilon})^2} \leq \frac{\mathrm{Var}(X)}{\frac{1}{2}\mathbb{E}(X)^2}$$

For $s \in S$, let A_s be the event "s is not in R" so that $X = \sum_{s\in S} \mathbf{1}_{A_s}$. Note $q = \mathbb{P}(A_s)$, which is independent of the choice of s in S. Note that the number of triangular relations containing s is an^2 with $a \sim 24$, so

$$q = (1-p_n)^{an^2} \geq \left(1 - \frac{(1-\varepsilon)\log n}{50n^2}\right)^{24.5n^2} \geq n^{-\frac{1-\varepsilon}{2}}$$

and

$$\mathbb{E}(X) = nq \geq n^{\frac{1+\varepsilon}{2}} \to \infty$$

The events $(A_s)_{s\in S}$ are not independent and X does not have a binomial distribution. In fact, for s, t different in S

$$\mathrm{Cov}(\mathbf{1}_{A_s}, \mathbf{1}_{A_t}) = q^2(1 - (1-p_n)^{-bn})$$

with $b \sim 48$, so

$$\frac{\mathrm{Var}(X)}{\mathbb{E}(X)^2} = \frac{nq(1-q) + n(n-1)q^2(1-(1-p_n)^{-bn})}{n^2q^2} \to 0$$

Which means that

$$\mathbb{P}(X < n^{\varepsilon}) \to 0$$

while $n \to \infty$. □

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Special cube complexes and generalised Bestvina-Brady Groups

Bestvina-Brady Morse theory is a great source of groups with interesting finiteness properties. Bestvina-Brady groups are subgroups of Right-Angled Artin Groups and have classifying spaces obtained from quotients of universal covers of Salvetti complexes which are special cube complexes. In [2], this construction was generalised by taking branched covers to give uncountably many more such groups, all of which are not subgroups of RAAGs.

My research has mostly concerned studying these generalised Bestvina-Brady groups. I am determining which ones contain the fundamental group of a special cube complex as a finite-index subgroup. More broadly I also think about residual finiteness and torsion-free finite-index subgroups in wider contexts [4].

One of the tools I have worked on is reducing the problem of whether a quotient complex is special to the representation theory of a finite quotient of the group. So even though I mostly think about non-finitely presented groups, I have also recently thought about constructing interesting finite groups.

In light of Agol's theorem, a lot of recent work on special cube complexes has come from hyperbolic groups. The complexes I work with contain many flats, and can give rise to infinite families of virtually special groups [3] coming from a non-hyperbolic context.

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Algebraic structure of groups from a geometric perspective

From September 2019 to February 2020, I pursued a master thesis at the Swiss Federal Institute of Technology of Lausanne, entitled "Some topics in the theory of hyperbolic groups", in which I studied different characterizations of Gromov hyperbolicity. In particular, I studied actions of groups on spaces with negative curvature, and I listed various families of hyperbolic groups coming from different backgrounds such as small cancellation theory. In september 2019, I started a PhD at the Heriot-Watt university, Scotland, under the supervision of Alexandre Martin. My first goal is to focus on actions on CAT(0) spaces and on the study of Coxeter and Artin groups.

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Asymptotic behaviour of locally symmetric spaces and harmonic analysis

Let G be a semisimple Lie group with a maximal compact subgroup $K \leq G$ and a sequence of lattices $\Gamma_n \leq G$. What can be said about the locally symmetric space $M_n = \Gamma_n \backslash G/K$ at the asymptotic level? For example, if $G = SL_2(\mathbb{R})$ then the covolume of Γ_n completely determines the topology of M_n due to Gauss-Bonnet. In higher dimensions, and in particular in higher rank where rigidity plays a role, the asymptotic behaviour can get much more complicated, and yet many interesting things could be said regarding topology (e.g the growth of Betti numbers), geometry (e.g systolic bounds) and harmonics (e.g the spectrum of the Laplace operator on M_n). Such concerns are related to many other topics which I'm interested in such as L^2-invariance, quantum ergodicity, and even quantum computing.

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Local subgroup properties of abstract commensurators

The abstract commensurator of a group G, denoted $\mathrm{Comm}(G)$, is the group of equivalence classes of isomorphisms $\phi : H \to K$, where H and K are finite index subgroups of G, under composition. We can think of the abstract commensurator as a natural generalization of the automorphism group $\mathrm{Aut}(G)$. Abstract commensurators arise naturally as a result of questions in hyperbolic geometry. Of particular interest are abstract commensurators of surface groups, such as $\mathrm{Comm}(F_2)$. I am interested in the local subgroup structure of $\mathrm{Comm}(F_2)$ as a way to consider the whole group.

Work by Bartholdi and Bogopolski [1] shows that $\mathrm{Comm}(F_2)$ is not finitely generated, and $\mathrm{Comm}(F_2)$ is known to contain embeddings of all finite groups and mapping class groups. Recent work by myself and Khalid Bou-Rabee (CCNY) has developed the notion of a abstract p-commensurator group, $\mathrm{Comm}_p(G)$, which embeds in $\mathrm{Comm}(F_2)$. This group provides a natural subgroup family for the study of more complicated abstract commensurators.

For integers m, n, the Baumslag-Solitar group $\mathrm{BS}(m, n)$ is given by the presentation

$$\langle a, b \mid ba^m b^{-1} = a^n \rangle.$$

My recent work with Bou-Rabee develops a natural family of images of the Baumslag-Solitar groups in $\mathrm{Comm}(F_2)$, and shows that these are not locally residually finite. A natural question is if $\mathrm{Comm}(F_2)$ contains embeddings of the Baumslag-Solitar groups.

Work of Bou-Rabee and Studenmund develops a method of evaluating words as images in the abstract commensurator [2], which Bou-Rabee and I have recently extended. This gives a new technique for handling groups with unsolvable word problem, and so I am also interested in images or embeddings of other infinite groups in $\mathrm{Comm}(F_2)$.

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Hyperbolic(-ish) subgroups in higher rank

I work in higher Teichmüller theory, which studies discrete subgroups of higher-rank semisimple Lie groups with good geometric and dynamical properties, including, usually, word-hyperbolicity, or possibly a controlled weakening thereof. Surface groups are prototypical examples of hyperbolic groups, and a lot of their geometry can be seen by viewing them as isometry groups of closed hyperbolic surfaces, which are discrete subgroups of the rank-one group $\mathrm{SL}(2,\mathbb{R}) \cong \mathrm{Isom}(\mathbb{H}^2)$; Teichmüller theory may be described as the study of how a surface group may be represented as such an isometry group. Free groups are also prototypical examples of hyperbolic groups, and they may be viewed as isometry groups of hyperbolic surfaces with funnels and/or cusps. The isometry group of a hyperbolic surface with funnels (but no cusps) is an example of a convex cocompact subgroup of $\mathrm{PSL}(2,\mathbb{R})$; the isometry group of a hyperbolic surface with cusps (and possibly also funnels) is an example of a geometrically finite subgroup. One can define convex cocompact subgroups of any semisimple Lie group G, although this is a less interesting concept when G has higher rank: Kleiner–Leeb [1] and Quint [2] proved that any irreducible convex cocompact subgroup in a higher rank semisimple Lie group G must be a uniform lattice. Alternatively, there is the class of Anosov subgroups, which coincides with the class of convex cocompact subgroups in rank one, and in higher rank constitutes a class of word-hyperbolic discrete subgroups which are quasi-isometrically embedded in the ambient Lie group, and which are structurally stable, i.e. small perturbations of Anosov subgroups remain Anosov. The Anosov condition admits many equivalent definitions: it can be approached from the geometry of the symmetric space of G [3], from the Lie theory associated with this geometry [4], or from hyperbolic dynamics [5][6]. Some of these definitions include word-hyperbolicity in the assumptions, but some of them allow one to deduce hyperbolicity as a consequence, usually via the dynamical characterization of a hyperbolic group acting on its boundary as a uniform convergence group. My work so far has concentrated on developing a notion of "relatively Anosov subgroup" (complementary to the work of Kapovich–Leeb in [7]) which might serve as a higher-rank analogue of geometric finiteness the same way that Anosov subgroups are higher-rank analogues of rank-one convex cocompact subgroups; these are relatively hyperbolic groups which sit in the ambient semisimple Lie group in a way which respects their intrinsic relatively hyperbolic geometry. I am interested in developing this notion further, and in exploring other notions of weakened hyperbolicity, such as acylindrical or hierarchical hyperbolicity, and how they may be useful (or not) in higher Teichmüller theory. I am also curious about the possibility of other "nice" classes of finitely-generated groups with "good" geometric and dynamical properties, broadly interpreted, which might show up as discrete subgroups of semisimple Lie groups.

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